INFLUENCE OF NUCLEAR MULTIPLE SCATTERING ON AXIALLY CHANNELED PROTONS IN A BENT CRYSTAL

by

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The influence of nuclear multiple scattering on axially channeled protons with an energy of 7 TeV through a bent <100> Si crystal, is presented in this paper. The aims of the investigation are the processes correlated to the axial channeling, such as dechanneling, angular distributions and energy loss distribution. The data for these processes are generated via the numerical solution of the proton equations of motion in the transverse plane and the computer simulation method. In the simulations, the crystal thickness is varied from 1 to 5 mm while the bending angle is varied from 0 to 20 rad. The increasing of the transverse energy of axially channeled protons is due to its multiple scattering by atomic strings and the bending dechanneling mechanism. The analysis of the generated data shows that in the cases we are considering, the dechanneling function, the energy loss spectra, and the angular distributions do not undergo to any significant changes when the effect of nuclear multiple scattering is included in the ion-atom interactions.

Key words: relativistic channeling, bent crystal, energy loss distribution, dechanneling function

INTRODUCTION

The ion-atom interaction in a crystal, which is oriented with respect to the atomic rows or planes, differs relative to the randomly oriented crystal. The main reason for this is the specific motion of the ions into the space between atomic strings (or planes), where the local electron density is small [1]. Because of this, the probability for transmitting of the ions through the crystal is high. This penetration of the ions trough the crystal is known as channeling.

One of the first experimental evidences of this effect was found in the energy loss distributions of protons in a short straight crystal of silicon and germanium [2].

The possibility of steering the high energy ions trough the bent crystal was predicted by Tsyganov [3]. First, his idea was proven in the computer simulation of Taratin [4], and soon after that, it vas experimentally confirmed by Elishev [5]. In the meantime, various aspects of ions channeling trough the bent crystals, such as angular distributions, energy loss distributions, beam collimation and beam deflection were investigated experimentally, theoretically, and by using the method of computer simulation [6-16].

During the passage of an ion through a crystal, as a result of ion-atom interaction, two mechanisms based on Coulomb multiple scattering are responsible for increasing of the ion transverse energy. The first one is the interaction of ion with crystal's nuclei and the second one is its interaction with electrons of the crystal.

Increasing of the ion transverse energy causes the increasing of the ion's scattering angle. When this angle is larger than some critical value, the ion is dechanneled, so the number of channeled ions is decreasing. The dechanneling investigation is based on the Kitagawa theory [17], and has been well studied in the case of planar channeling [18-20].

The possibility of using the bent crystal as a tool for beam collimation in the Hadron colliders, have initiated an investigation of the elastic and inelastic scattering of the ions by the nuclei of the crystals [21-24].

In this paper, based on the numerically generated data, we analyze the influence of the nuclear multiple scattering on the process of dechanneling, energy loss spectra and angular distributions of the axially channeled protons with energy of 7 TeV through a bent <100> crystal of silicon.

THEORY

In this paper, we use a model for axial channeling of ions through a bent crystal which takes into account the interactions of ions with the electrons and

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nuclei of the crystal. The continuum interaction potential of the ion and the atomic string is obtained from the Moliere's expression for the ion-atom interaction potential $U_i(r)$ [25-27]

$$U_i(r) \quad \frac{2Z_1Z_2e^2}{d} \quad 0.1K_0 \quad \frac{6r}{a_{\mathrm{TF}}} \quad 0.55K_0 \quad \frac{1.2r}{a_{\mathrm{TF}}} \quad 0.35K_0 \quad \frac{0.3r}{a_{\mathrm{TF}}} \tag{1}$$

where Z_1 and Z_2 are the atomic numbers of ion and atoms of the crystal, respectively, e is the elementary charge, d – the distance between the atoms in the atomic string, $a_{\mathrm{TF}} = 0.8853\ a_0\ (Z_2)^{-1/3}$ – the screening radius, a_0 is the Bohr radius, $r\ (x, y) = (x^2 + y^2)^{1/2}$ – the distance between the ion and the i-th atomic string, and K_0 – zero-order modified Bessel function of the second kind. The thermal vibrations of the crystal atoms are taken into account [26, 27] and the continuum interaction potential of the crystal is the sum of the continuum interaction potentials of the atomic strings, $U^{th} \quad U_i^{th}$.

In the case of channeling trough a bent crystal, the effective continuum interaction potential is given by the following expression

$$U_{eff}^{th}(x, y) \quad U^{th}(x, y) \quad \frac{p\upsilon}{R} q \tag{2}$$

where the substitutions $q = x$ or $q = y$ depends on the crystal bending direction [27], p is the relativistic momentum of the ion, υ – the corresponding velocity, and R – the radius of curvature of the crystal.

After entering the crystal, the ion transverse energy is increasing as a result of its interactions with atomic strings and the influence of the centrifugal force. It means that the average angle of the ion velocity vector ψ, in the transverse plane with respect to the crystal axis, increases too. When this angle is larger than the critical angle for axial channeling ψ_c, the ion is dechanneled and disregarded [27]. So, decreasing of the number of channeled ions in a bent crystal is a result of two dechanneling mechanisms: the first one is the multiple scattering from the crystal's electrons and the crystal's nuclei, and the second one is the bending dechanneling mechanism which originates from the crystal curvature [18-20].

It is well known that the process of dechanneling depends on (total) energy losses of channeled ions. In our model, the electronic energy loss of the ion was calculated using the expression for the trajectory dependent stopping power [19, 28]

$$\frac{\mathrm{d}E}{\mathrm{d}z} \quad \frac{D}{2\beta^2} \quad \ln\frac{2m_ec^2\beta^2\gamma^2}{I} \quad \beta^2 \quad \delta \quad \rho_e(x, y) \quad \ln\frac{T_{\max}}{I} \quad \frac{T_{\max}}{2m_ec^2\gamma^2} \tag{3}$$

where $D \quad 4\pi r_e^2 m_e c^2 Z_1^2 \langle n\rangle$, r_e is the classical radius of the electron, $\langle n\rangle \quad NZ_2$ – the averaged electron density, N – the crystal atom density, $\rho_e(x, y) \quad n_e(x, y)/\langle n\rangle$ is the normalized local electron density and the rest are standard notations [19, 29, 30].

The local electron density $n_e(x, y)$ averaged along z-direction is calculated by Poisson eq. [20, 27]

$$\Delta U^{th}(x, y) \quad 4\pi[Q_e(x, y) \quad Q_n(x, y)] \tag{4}$$

where $Q_e(x, y) = -en_e(x, y)$ is the local density of the negative (electron) electric charge and $Q_n(x, y) = Z_2 e\ n_n(x, y)$ – the local positive (nuclei) electric charge. For the local nuclei density $n_n(x, y)$ we use the expression [17, 27]

$$n_n(x, y) \quad \frac{L_d}{\pi N d\ \sigma_{th}^2} \exp \quad \frac{r_t^2}{\sigma_{th}^2} \tag{5}$$

where d and L_d are the distance between atoms and linear density of atoms in <100> axial direction of silicon, respectively, and $r_t(x, y)$ is the position of the ion in the transversal plane. The one-dimensional amplitude of the thermal vibration of silicon atoms σ_{th} is 0.00744 nm [31, 32].

The mean-square scattering angle of the ion per unit length, caused by its interaction with electrons of the crystal, is [20, 27]

$$\frac{\mathrm{d}\Omega_e^2}{\mathrm{d}z} \quad \frac{m_eD}{2p^2\beta^2} \quad \ln\frac{2m_ec^2\beta^2\gamma^2}{I} \quad \beta^2 \quad \delta \quad \rho_e(x, y) \tag{6}$$

while the corresponding uncertainties of the ion scattering angle, which is a result of its interaction with the crystal nuclei, is calculated by

$$\frac{\mathrm{d}\Omega_n^2}{\mathrm{d}z} \quad \frac{m_ec^2Z_2D}{\upsilon_0^2p^2} \ln\ 1.29\frac{a_{\mathrm{TF}}E_k}{Z_1Z_2\hbar c}\frac{m_2}{m_1 \quad m_2}\ n_n(x, y) \tag{7}$$

In this expression E_k and υ_0 are initial kinetic energy and speed of the ion, respectively, m_2 is the mass of the crystal nucleus, m_1 is the ion mass and the rest are already given notations [17, 20].

Taking into account the partial uncertainties of the ion scattering angle, the total mean-square scattering angle could be calculated by the expression

$$\frac{\mathrm{d}\Omega^2}{\mathrm{d}z} \quad \frac{\mathrm{d}\Omega_e^2}{\mathrm{d}z} \quad \frac{\mathrm{d}\Omega_n^2}{\mathrm{d}z} \tag{8}$$

where the transversal components of scattering angle Ω_x and Ω_y fulfill the expression $\Omega_x \quad \Omega_y \quad \Omega/\sqrt{2}$.

The simulated angular distributions, energy loss distributions and corresponding dechanneling functions of axially channeled protons, were obtained via the numerical solution of the ion's equations of motion in the transverse plane [16, 27] and the uncertainties of the ion scattering angle was calculated by the expression (8).

RESULTS AND DISCUSSION

We shall analyze here in details the influence of nuclear multiple scattering on axially channeled 7 TeV protons through a bent <100> silicon crystal. The distance between atoms d, along the <100> atomic string, is 0.543082 nm [33] and the critical angle of axial channeling ψ_c is 4.6 rad. The proton beam divergence, before its interaction with the crystal, was taken to be 0.46 rad.

The dechanneling of protons depending on the bending angle was investigated in the case of a crystal with constant thickness of L = 1 mm and a bending angle that varies from 5 to 20 rad. The initial number of the ions was N_p = 103,041 and their initial positions in the transversal plane of the channel were distributed uniformly. The ion whose initial position lies within the screening radius around the atomic strings defining the channel, was treated as backscattered and disregarded.

Figure 1(a) shows the dechanneling functions of the 7 TeV protons axially channeled through a bent <100> Si crystal when the nuclear multiple scattering is not taken into account. The simulated data can be excellently fitted with the Gompertz type function [34, 27]

$$\frac{N_d}{N_0} \quad \frac{\exp\{\ \exp[\ k(\alpha\ \ \alpha_c)]\}\ \ \exp[\ \ \exp(k\alpha_c)]}{1\ \ \exp[\ \ \exp(k\alpha_c)]} \quad (9)$$

where N_d is the number of dechanneled protons, N_0 = $= N - N_b$ – the initial number of protons without the backscattered ones, k – the dechanneling rate and α_c – the dechanneling range. The values of the best fitting parameters are $\alpha_c = 6.914$ rad and $k = 0.386$ rad^{-1}.

When the nuclear multiple scattering is on, the dechanneling function of axially channeled protons is shown in fig. 1(b). It is obvious, as in the previous case, that the simulated data can be fitted with the Gompertz type function and the best fitting parameters have almost identical values α_c = 6.915 rad and k = = 0.386 rad^{-1}. This means that the effect of nuclear multiple scattering does not affect the dechanneling process, when the thickness of the crystal is constant and the intensity of the centrifugal force increases by increasing the bending angle of the crystal.

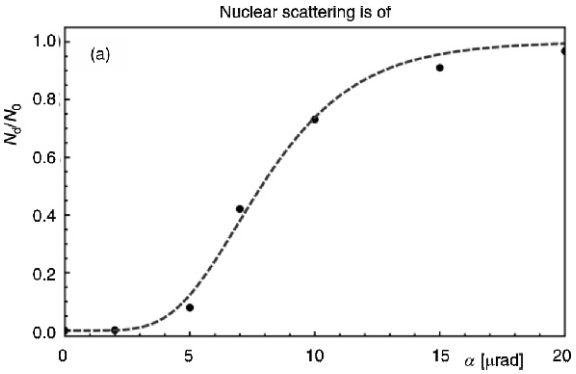

Figure 1(a). The dechanneling functions of the 7 TeV protons axially channeled through a bent <100> Si crystal. The thickness of the crystal is L = 1 mm and the curvature of the crystal is varied from 2 rad to 20 rad; the effect of nuclear multiple scattering is off

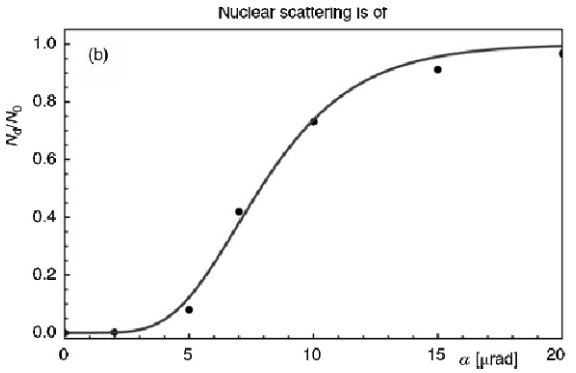

Figure 1(b). The dechanneling functions of the 7 TeV protons axially channeled through a bent <100> Si crystal; the input parameters are the same as in the fig. 1(a), but the effect of nuclear multiple scattering is on

Figure 2 presents the energy loss distribution of axially channeled protons for bending angle α = = 5 rad, when the nuclear multiple scattering is off [35]. For a better analysis of the data, the histogram is di-

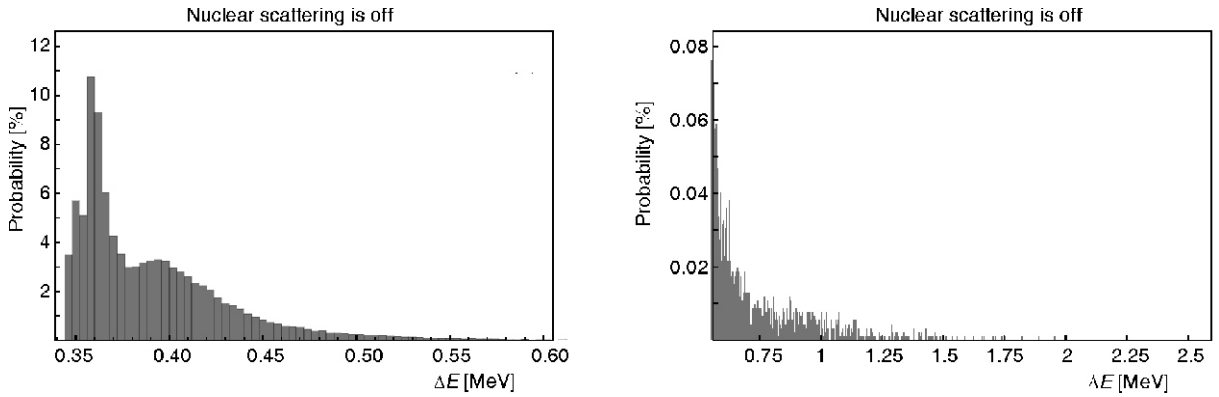

Figure 2. The energy loss distribution of 7 TeV protons axially channeled through a bent <100> silicon crystal when the effect of nuclear multiple scattering is off

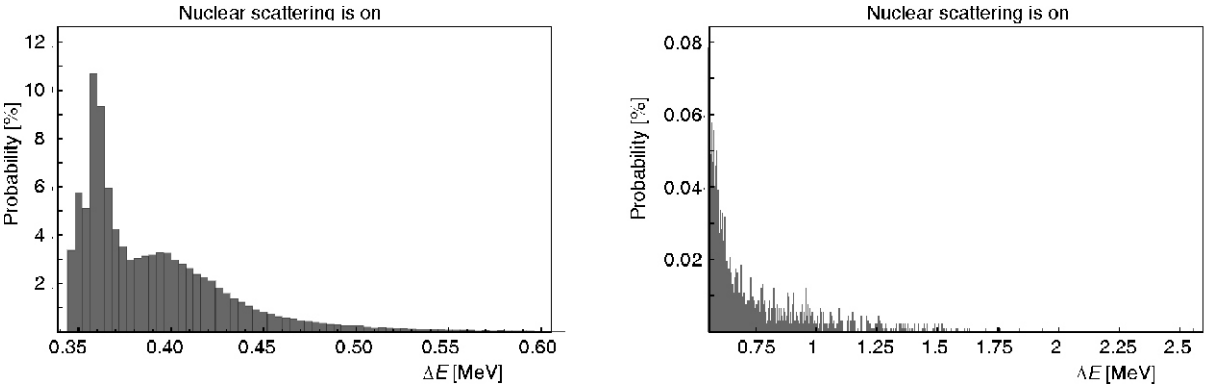

Figure 3. The energy loss distribution of 7 TeV protons axially channeled through a bent <100> silicon crystal when the effect of nuclear multiple scattering is on

vided into the two parts. The energy loss distribution, which originates from the ions located in between atomic strings and from the central area of the channel, is designated by (a), while the high energy tail of the energy loss distribution, which originates from the ions with initial positions in the vicinity of the atomic strings, is designated by (b).

Figure 3 shows the energy loss distribution of axially channeled protons for the same bending angle, $\alpha = 5$ rad, when the nuclear multiple scattering is on. It is obvious that this specific scattering mechanism of the protons, from the nuclei of the crystal, causes only extension of the high energy tail of ~31 %, till energy of $\Delta E < 2.6$ MeV, and minor "structural" modification of the energy loss distribution in the area 0.6 MeV $< \Delta E < 0.8$ MeV.

Further, we shall analyze the influence of the nuclear multiple scattering on the angular distributions of 7 TeV protons, axially channeled through a bent <100> Si crystal. In this case, the bending angle of the crystal is $\alpha = 5$ rad and its thickness is $L = 1$ mm. Figure 4(a) shows the angular distribution when the nuclear multiple scattering is off and fig. 4(b) shows the corresponding angular distribution when the nuclear multiple scattering is on. Comparing the fig. 4(a) and fig. 4(b), one can conclude that both distributions are very similar to each other and are characterized by a central maximum around zero angle and the two additional maxima close to the zero angle [12, 16]. The only visible differences between the angular distributions are small structural changes in the vicinity of the central maxima.

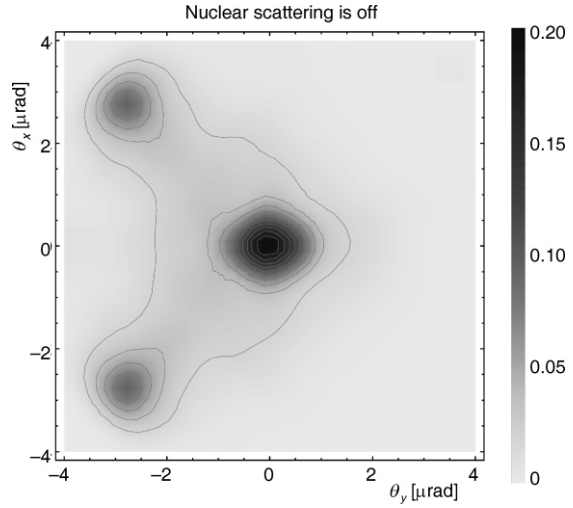

Figure 4(a). The angular distributions of the 7 TeV protons axially channeled through a bent <100> Si crystal; the thickness of the crystal is L = 1 mm and the curvature of the crystal is 5 μrad; the effect of nuclear multiple scattering is off

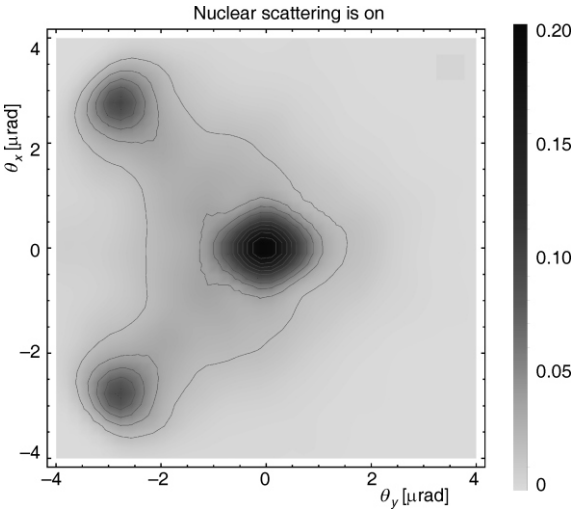

Figure 4(b). The angular distributions of the 7 TeV protons axially channeled through a bent <100> Si crystal; the thickness of the crystal is L= 1mm and the curvature of the crystal is 5 μrad; the effect of nuclear multiple scattering is on

Dependence of the number of dechanneled protons on the crystal thickness, or so called the dechanneling function, is presented in fig. 5. The analysis shows that simulated data can be fitted with the Gompertz type function (9). In the case when the nuclear multiple scattering is on, the dechanneling rate has a value $k = 1.724$ mm^{-1}, whereas the dechanneling range is $\alpha_c = 0.994$ mm.

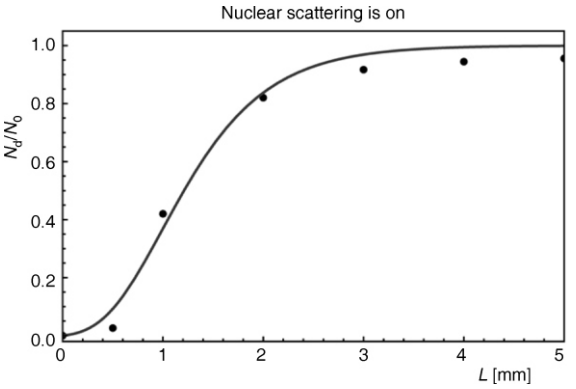

Figure 5. The dechanneling functions of the 7 TeV protons axially channeled through a bent <100> Si crystal; the bending angle of the crystal is = 5 μrad, and its thickness is varied from 1 to 5 mm; the effect of nuclear multiple scattering is on

CONCLUSIONS

In this paper we have shown that in the case of axial channeling of high energy protons through a bent crystal with a constant thickness, L = 1mm, the process of protons dechanneling, energy loss distribution and the angular distribution, do not undergo to any significant changes when the nuclear multiple scattering is taken into account. The main reason for this is the fact that the area around the atomic strings, defined by the screening radius, is very much smaller than the area of the channel, which means that the effect of nuclear multiple scattering is not probable and can be neglected under the axial channeling conditions.

The only significant change due to the effect of the nuclear multiple scattering is an extension of the high energy tail for around 31 %, in the case of a bent <100> crystal of silicon. It means that the effect of nuclear multiple scattering occurs rarely, but it causes more intense energy losses of the channeled protons than multiple scattering by the electrons of the crystal.

The next important result is the estimation of the dechanneling range of 7 TeV protons under axial channeling condition. For this purpose the bending angle of the crystal is α = 5 rad, while its thickness is varied from 1 to 5 mm. The analysis has shown that, the calculated dechanneling function can be excellently fitted by the Gompertz type function (9). The best fitting values for dechanneling rate and dechanneling range are k = 1.724 mm^{-1} and α_c = 0.994 mm, respectively.

AUTHORS' CONTRIBUTIONS

Theoretical analysis was carried out by N. Stojanov, S. Petrović and D. Jakimovski. All authors analyzed and discussed the results. The manuscript and figures were prepared by N. Stojanov.

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УТИЦАЈ ВИШЕСТРУКОГ НУКЛЕАРНОГ РАСЕЈАЊА НА АКСИЈАЛНО КАНАЛИСАНЕ ПРОТОНЕ У ЗАКРИВЉЕНОМ КРИСТАЛУ

У овом раду приказан је утицај вишеструког нуклеарног расејања на аксијално каналисане протоне са енергијом од 7 TeV кроз <100> савијеног Si кристала. Циљеви истраживања су процеси који су у корелацији са аксијалним каналисањем, као што су деканалисање, угаоне расподеле и расподеле енергетских губитака јона. Подаци за ове процесе генеришу се путем нумеричког решења једначина кретања протона у трансверзалној равни и методе компјутерске симулације. У симулацијама дебљина кристала варира од 1 mm до 5 mm, а угао савијања варира од 0 rad до 20 rad. Повећање трансверзалне енергије аксијално каналисаних протона је резултат њиховог вишеструког расејања са атомским низовима и механизмом деканалисања због закривљења кристала. Анализа генерисаних података показује да у случајевима које разматрамо, функција деканалисања, спектар енергетских губитака и угаоне расподеле не подлежу никаквим значајним променама када је у интеракције јона и атома укључен и ефекат вишеструког нуклеарног расејања.

Кључне речи: релативистичко каналисање, закривљени кристал, расподела енергијског губитка, функција деканалисања