

## NUMERICAL SIMULATION OF SHOCK ABSORBERS HEAT LOAD FOR SEMI-ACTIVE VEHICLE SUSPENSION SYSTEM

by

**Miroslav D. DEMIĆ<sup>a</sup> and Djordje M. DILIGENSKI<sup>b\*</sup>**

<sup>a</sup> Faculty of Engineering Sciences, University of Kragujevac, Kragujevac, Serbia

<sup>b</sup> Department for Engines and Vehicles, Vinča Institute of Nuclear Sciences, University of Belgrade, Belgrade, Serbia

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*Dynamic simulation, based on modelling, has a significant role during the process of vehicle development. It is especially important in the first design stages, when relevant parameters are to be defined. Shock absorber, as an executive part of a semi-active suspension system, is exposed to thermal loads which can lead to its damage and degradation of characteristics. Therefore, this paper attempts to analyse a conversion of mechanical work into heat energy by use of a method of dynamic simulation. The issue of heat dissipation from the shock absorber has not been taken into consideration.*

Key words: *vehicle, vibration parameters, degradation, thermal load of shock absorber*

### Introduction

Shock absorbers are an integral part of the semi-active suspension system, which directly affect the active vehicle safety. The role of shock absorber in addition to its basic, is to absorb mechanical vibrations transferred from the ground and consecutively to ensure the safety of passengers in a vehicle.

As it is known, in shock absorbers the kinetic energy of the system transforms into mechanical work or heat [1-6]. In practice, in the stage of vehicle development, shock absorber parameters are chosen from the condition of damping of vehicle vibrations, but in order to avoid the negative impact on the function, thermal loads should be taken into consideration [4]. The goal is to convert, as much as possible, the mechanical work received from the ground into thermal energy which will be transferred to the environment and thus provide the cooling of shock absorber. Wrong selection of shock absorbers, from the standpoint of thermal loads, can cause rapid degradation of its properties during service life. Excessive amount of heat energy, eventually kept "inside" the shock absorber, would cause rapid deterioration of sealing elements and loss of function of the damping element.

Tests have shown that the mechanical work is partly converted into the heat which is transferred to the working fluid and dampers body, and the remaining amount of heat delivery is transferred to the environment, thus cooling the shock absorber. Mathematically, it can be displayed by the formula [3, 7-11]:

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\* Corresponding author; e-mail: diligens@vinca.rs

$$A = Q_t + Q_f + Q_e \quad (1)$$

where  $A$  is the mechanical work (equal to the quantity of heat),  $Q_t$  – the quantity of heat transferred to the shock absorbers body,  $Q_f$  – the quantity of heat transferred to the working fluid, and  $Q_e$  – the quantity of heat transferred to the environment.

The work of the force in the shock absorber is of relevance because it enables the analysis of its transformation into heat energy. The work of force in the shock absorber is experimentally measurable, but it is hard to measure the amount of heat released from the shock absorber [12]. This phenomenon is complex and difficult to measure because it is known that a part of the energy is consumed to heat shock absorbers elements, such as piston, cylinder, working fluid, *etc.* In addition, the nature of heat transfer from the absorber to the environment is very complex. Heat transfer is carried out by convection, as dominant, also by conduction and radiation [4].

From the point of maximal cooling, proper selection of shock absorbers requires a comprehensive analysis of the transformation of mechanical energy into heat. Method of transformation of mechanical energy into heat is largely determined by the shock absorbers design. It is not possible to influence directly on the conduction of heat and radiation from the shock absorber. It is necessary to increase the influence of the heat transfer by convection from the shock absorber to the surrounding environment, as dominant appearance. This is best achieved by design of the vehicle body (fender). The idea is to utilize convection flow of air around the absorber with the least complexity of the structure. In practice, this solution is rarely used, but can be applied. Making some kind of air deflectors on the elements of the body should increase the effect of convective heat transfer to the environment.

Note that the objective of this study was not to analyse the cooling of shock absorber, but only thermal load to which it is exposed. Therefore, it was deemed expedient to analyse the heat which is obtained by converting mechanical work into heat energy per time unit. Mechanical work in shock absorbers of a semi-active suspension system was calculated by use of mechanical model of the vehicle, which will be discussed below.

### Vehicle model with semi-active suspension system

Motor vehicles have complex dynamic characteristics, which are displayed by the appearance of spatial movements, variation of characteristics during operation, a number of disturbing influences, the appearance of clearances, friction, hysteresis, *etc.* [1, 2, 13-21]. The above mentioned dynamic phenomena, especially vibrations, lead to the fatigue of passengers and driver, reduce service life of the vehicle and its systems, *etc.* The main objective of the suspension system is to reduce the above mentioned negative effects, improving the vehicle's behaviour on the road and to allow the exploitation of the vehicle in a wide range of service conditions.

Classical systems cannot meet the above-mentioned requirements, so there was a need to introduce new suspension system with controlled characteristics (briefly called "semi-active", or "active" systems). Their classifications are known from [20, 22], and it will not be discussed here.

The fact is that classic "passive" systems can only offer a compromise between the conflicting demands for comfort and vibration behaviour of the vehicle on the road [1, 2, 13-21, 23, 24], because their stiffness and damping parameters are not variable. Given the tendency for lower fuel consumption, especially in the midsize vehicle segment, which, among other options, can be achieved by the reduction on the vehicle curb weight, it is obvious that the as-

pect of “loaded/unloaded” condition manifests degradation of parameters of ride comfort and vehicle handling performances on the road.

Hence it can be concluded that only the suspension systems with controlled properties can satisfactorily compensate for disturbing influences, and historically observed, this system was first used in the cosmic ships (“Sputnik”, between 1950 and 1960, while the application in ground vehicles begins in the 1970s [25] and the first with the “semi-active” concept.

In practice, there are two kinds of systems with controlled characteristics [17, 21, 25-34]:

- (1) “*semi-active*”, in which the control of stiffness or damping is being carried out; they have favourable characteristics, and their price is acceptable for vehicles of lower categories, and
- (2) “*active*”, with power generator (actuator) that can be combined with conventional elasto-damping elements; their price is very high, so the application is for high-class vehicles or special vehicles only.

Since there are different design aspects of these systems, which are discussed in details in [25], this paper will discuss the semi-active suspension systems with the adjustable damping properties of shock absorbers and their thermal load. Besides, the impact of variation of degradation of shock absorbers properties on thermal characteristics, due to aging, will be observed.

### Vehicle vibration model

Bearing in mind the research goal, the following secondary (vibration) vehicle mass motions were subjected to observation (fig. 1):

- |                           |                                    |
|---------------------------|------------------------------------|
| – Sprung mass             | – Unsprung mass vibration (wheels) |
| $z$ – vertical vibration, | $z_{fl}$ – front left,             |
| $\varphi$ – roll,         | $z_{fr}$ – front right,            |
| $\theta$ – pitch          | $z_{rl}$ – rear left,              |
|                           | $z_{rr}$ – rear right              |

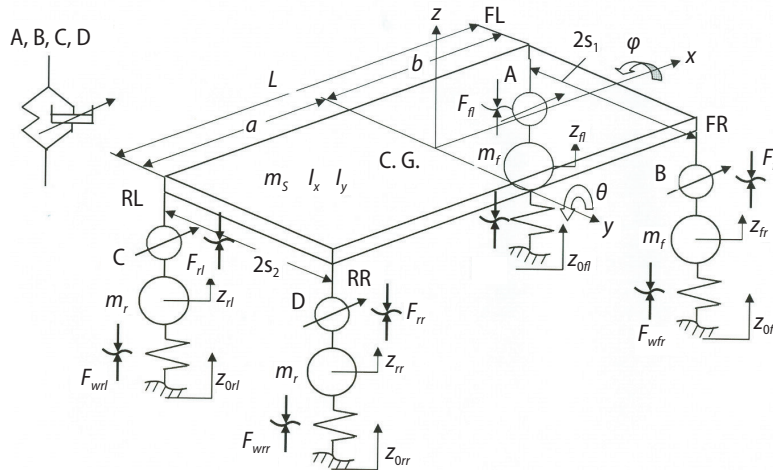
Differential equations of the vibratory motion of the observed vehicle model are written by use of software package NEWEUL [35]. The preparation of input data was carried out in accordance with the requirements of the mentioned software. For definition of dynamic vehicle model with 7 degrees of freedom, 24 coordinate systems were used (including inertial-global co-ordinate system  $OI_1I_2I_3$ ), with reference to which the inertial parameters, forces, etc. were specified accordingly. Since the procedure of the input data preparation is thoroughly described in [35, 36], it will not be repeated here.

It should be noticed that generalized co-ordinates of the sprung mass ( $z$ ,  $\varphi$ , and  $\theta$ ) and generalized co-ordinates of wheels masses vibration ( $z_{fl}$ ,  $z_{fr}$ ,  $z_{rl}$ , and  $z_{rr}$ ) are defined the reference to their equilibrium position. Therefore, gravity forces of the observed masses are not included in differential equations of the vehicle motion [1, 2, 13-21, 24]. Non-linear differential equations of vehicle motion (fig. 1) were written by use of software NEWEUL [35, 36]:

$$M\ddot{q} + K = QE \tag{2}$$

where  $M$  is the matrix of inertial parameters (DOF×DOF – DOF number of degrees of freedom),  $K$  – the vector of Coriolis and centrifugal forces (1×DOF),  $QE$  – the vector of generalized forces, and  $\ddot{q}$  – vector of generalized accelerations (1×DOF).

Inertial parameters of the observed vehicle model are defined by:  $G_s$ ,  $m_s$ ,  $m_f$ ,  $m_r$  – gravitational force and sprung mass, unsprung masses of front and rear wheel systems, respectively,  $I_x$  – the sprung mass moment of inertia for x-axis,  $I_y$  – the sprung mass moment of inertia for y-axis, and  $I_z$  – the sprung mass moment of inertia for z-axis.



**Figure 1. Space model of a vehicle with semi-active suspension system**  
(A, B, C, D system of springs with fixed parameters and shock absorbers with controlled damping)

In case of the observed vehicle space model, differential equations describe vibration motions in the following form:

$$\begin{aligned}\ddot{z} &= (QE[1] - K[1])/m_s; \quad \ddot{\varphi} = (QE[2] - K[2])/(I_x \cos^2 \theta + 2I_z \sin^2 \theta); \\ \ddot{\theta} &= (QE[3] - K[3])/I_y;\end{aligned}\tag{3}$$

$$\ddot{z}_{fl} = (QE[4] - K[4])/m_f; \quad \ddot{z}_{fr} = (QE[5] - K[5])/m_f;$$

$$\ddot{z}_{rl} = (QE[6] - K[6])/m_r; \quad \ddot{z}_{rr} = (QE[7] - K[7])/m_r$$

Vectors  $K$ , and  $QE$  are given by expressions (see eq. 2):

$$\begin{aligned}K[1] &= 0; \quad K[2] = 2(I_z - I_x)\dot{\varphi}\dot{\theta}\sin(\theta)\cos(\theta); \quad K[3] = (I_x - I_z)\dot{\varphi}^2\sin(\theta)\cos(\theta); \\ K[4] &= 0; \quad K[5] = 0; \quad K[6] = 0; \quad K[7] = 0\end{aligned}\tag{4}$$

$$\begin{aligned}QE[1] &= -(F_{fl} + F_{fr} + F_{rl} + F_{rr})\cos(\varphi)\cos(\theta); \\ QE[2] &= (F_{fl} - F_{fr})s_1\cos(\varphi)\cos(\theta) + (F_{rl} - F_{rr})s_2\cos(\varphi)\cos(\theta); \\ QE[3] &= (F_{fl} + F_{fr})a\cos(\theta) + (F_{rr} - F_{rl})s_1\sin(\varphi)\sin(\theta) - (F_{fl} + F_{fr})b\cos(\theta) + \\ &+ (F_{rr} - F_{rl})s_2\sin(\varphi)\sin(\theta); \\ QE[4] &= -F_{wfl} + F_{fl}\cos(\varphi)\cos(\theta); \quad QE[5] = -F_{wfr} + F_{fr}\cos(\varphi)\cos(\theta); \\ QE[6] &= -F_{wrl} + F_{rl}\cos(\varphi)\cos(\theta); \quad QE[7] = -F_{wrr} + F_{rr}\cos(\varphi)\cos(\theta)\end{aligned}\tag{5}$$

Displacements of characteristic points, relevant for calculation of collective forces in springs and shock absorbers ( $F_{fl}$ ,  $F_{fr}$ ,  $F_{rl}$  and  $F_{rr}$ ), are automatically calculated by the same software. They are given by expressions:

$$\begin{aligned} \Delta_{fl}, \Delta_{fr} &= a \sin(\theta) \pm s_1 \sin(\varphi) \cos(\theta) + z \cos(\varphi) \cos(\theta) \\ \Delta_{rl}, \Delta_{rr} &= -b \sin(\theta) \pm s_1 \sin(\varphi) \cos(\theta) + z \cos(\varphi) \cos(\theta) \end{aligned} \quad (6)$$

Relative piston displacements in shock absorbers can now be calculated:

$$\Delta_{fl}^* = \Delta_{fl} - z_{fl}; \quad \Delta_{fr}^* = \Delta_{fr} - z_{fr}; \quad \Delta_{rl}^* = \Delta_{rl} - z_{rl}; \quad \Delta_{rr}^* = \Delta_{rr} - z_{rr} \quad (7)$$

Velocities of motion ( $\dot{\Delta}_{fl}^*, \dot{\Delta}_{fr}^*, \dot{\Delta}_{rl}^*, \dot{\Delta}_{rr}^*$ ) can be calculated as time derivatives of the expression (6), what will not be done here.

Forces in shock absorbers with uncontrolled characteristics depend on relative velocities of the piston related to the shock absorbers body and the corresponding coefficients, and can be described with the following expressions:

$$F_{ai} = x[15,19] \cdot (\dot{\Delta}_i - \dot{z}_i) + x[16,20] \cdot (\dot{\Delta}_i - \dot{z}_i)^2 \text{sign}(\dot{\Delta}_i - \dot{z}_i) \quad (8)$$

where  $i = fl, fr, rl, \text{ and } rr$ ,  $x[15]$ ,  $x[19]$ ,  $x[16]$ , and  $x[20]$  are shock absorbers damping parameters.

In case of semi-active suspension system, values  $x[15]$ ,  $x[19]$ ,  $x[16]$ , and  $x[20]$  are subject to control and will be discussed below in more details.

For easier understanding of the following text, fig. 2 will be observed, showing control scheme of semi-active suspension system of the observed vehicle model. In the specific case, a concept of control of the suspension system is realized by means of two groups of feedback reactions. The external feedbacks are assigned to minimize the influence of road microroughness, and internal to control forces in shock absorbers. To be more specific, external feedback loops are to control ride comfort and internal feedback should enable favourable load distribution and levelling, in case of vehicle maneuver (the aspect of vehicle handling). It is obvious from

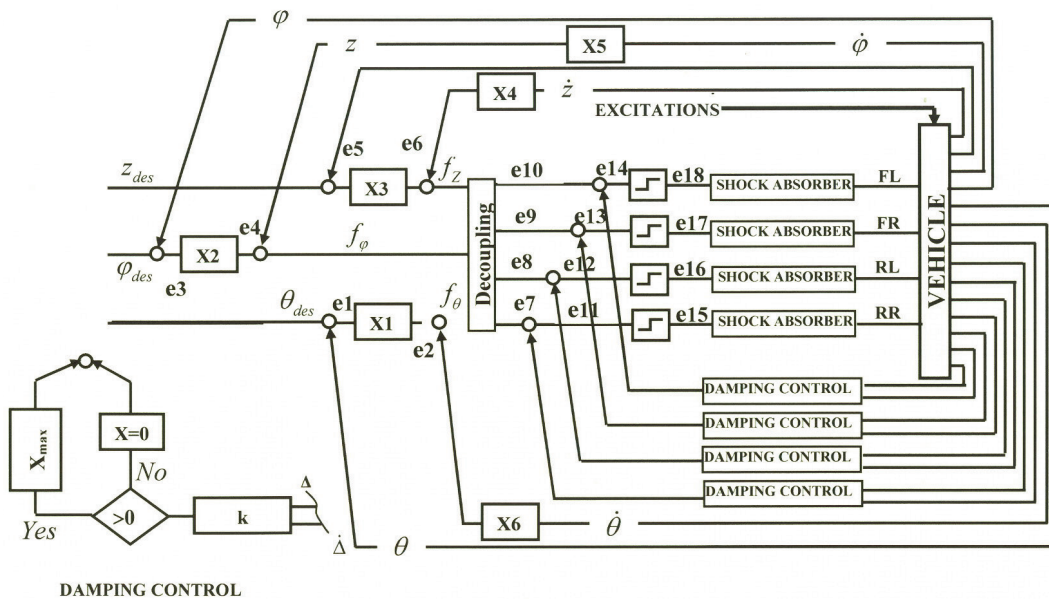


Figure 2. Block scheme of the observed semi-active suspension system

fig. 2 that this concept of semi-active suspension is based on recording of sprung mass vertical, roll and pitch vibration, as well as cylinder piston displacements and corresponding velocities.

Since the forces in elasto-damping elements (sum of forces in spring and shock absorber)  $F_{fb}$ ,  $F_{fr}$ ,  $F_{rl}$  and  $F_{rr}$  depend on sprung mass vertical, roll and pitch vibration, the components causing vertical vibration  $z$  should be separated from those inducing roll- $\varphi$  and pitch- $\theta$  (see block "Decoupling" in fig. 2). Based on generalized forces  $F_z=QE[1]$ ,  $F_\varphi=QE[2]$  and  $F_\theta=QE[3]$  (eq. 5), together with generalized co-ordinates, the relations that link resulting signals  $f_z$ ,  $f_\varphi$  and  $f_\theta$  with the parts of force control signals in shock absorbers  $f_{fb}$ ,  $f_{fr}$ ,  $f_{rl}$  and  $f_{rr}$  can be written in the following form:

$$\begin{bmatrix} f_z \\ f_\varphi \\ f_\theta \end{bmatrix} = \begin{bmatrix} -c\varphi c\theta & c\varphi c\theta & -c\varphi c\theta & -c\varphi c\theta \\ s_1 c\varphi c\theta & -s_1 c\varphi c\theta & s_2 c\varphi c\theta & -s_2 c\varphi c\theta \\ a c\theta - s_1 s\varphi s\theta & a c\theta + s_1 s\varphi s\theta & -b c\theta - s_2 s\varphi s\theta & -b c\theta + s_2 s\varphi s\theta \end{bmatrix} \begin{bmatrix} f_{pl} \\ f_{pd} \\ f_{zl} \\ f_{zd} \end{bmatrix} \quad (9)$$

where abbreviations  $s(\cdot)=\sin(\cdot)$  and  $c(\cdot)=\cos(\cdot)$  were introduced for the sake of brevity, and matrix dimensions are  $4 \times 3$ . It should be noticed that eq. (9) includes only forces in shock absorbers, whose parameters are controlled, while spring parameters are fixed values.

The expression (10) can be written shortly, in matrix form:

$$F = AF_F \quad (10)$$

where from the relation follows:

$$F_F = A^{-1}F \quad (11)$$

The procedure for calculation of pseudo-inverse matrix  $A^{-1}$  is known from linear algebra:

$$A^{-1} = A^T (AA^T)^{-1} \quad (12)$$

where  $A^T$  is transposed matrix of matrix  $A$ . Bearing in mind the complexity of matrix  $A$  (expression (7)), a pseudo-inverse matrix is calculated by use of program MATHEMATICA, leading to the expression:

$$\begin{bmatrix} f_{fb} \\ f_{fr} \\ f_{rl} \\ f_{rr} \end{bmatrix} = \begin{bmatrix} \frac{b}{Lc\varphi c\theta} & \frac{(s_1^2 + s_2^2)s\varphi s\theta + s_1 Lc\theta}{2L(s_1^2 + s_2^2)c\varphi c^2\theta} & \frac{1}{2Lc\theta} \\ \frac{b}{Lc\varphi c\theta} & \frac{(s_1^2 + s_2^2)s\varphi s\theta - s_1 Lc\theta}{2L(s_1^2 + s_2^2)c\varphi c^2\theta} & \frac{1}{2Lc\theta} \\ \frac{a}{Lc\varphi c\theta} & -\frac{(s_1^2 + s_2^2)s\varphi s\theta - s_2 Lc\theta}{2L(s_1^2 + s_2^2)c\varphi c^2\theta} & -\frac{1}{2Lc\theta} \\ \frac{a}{Lc\varphi c\theta} & -\frac{(s_1^2 + s_2^2)s\varphi s\theta + s_2 Lc\theta}{2L(s_1^2 + s_2^2)c\varphi c^2\theta} & -\frac{1}{2Lc\theta} \end{bmatrix} \cdot \begin{bmatrix} f_z \\ f_\varphi \\ f_\theta \end{bmatrix} \quad (13)$$

where  $f_{fb}$ ,  $f_{fr}$ ,  $f_{rl}$  and  $f_{rr}$  ( $e7$ ,  $e8$ ,  $e9$  and  $e10$ ) refer to control signals for the forces in shock absorbers. Values  $e7$ ,  $e8$ ,  $e9$ , and  $e10$  are representing those  $f$  signals in the block scheme presented in fig. 2.

Based on the adopted concept of force control in shock absorbers and fig. 2, the following relations can be written:



$$\begin{aligned}
 e1 &= \theta_{des} - \theta; \quad e2 = e1x[1]; \quad e3 = \varphi_{des} - \varphi; \quad e4 = e3x[2]; \quad e5 = z_{des} - z; \quad e6 = e3x[2]; \\
 f_\varphi &= e2 - x[6]\dot{\varphi}; \quad f_\theta = e4 - x[5]\dot{\theta}; \quad f_z = e5 - x[4]\dot{z}; \quad e7 = f_{rr}; \quad e8 = f_{rl}; \quad e9 = f_{fr}; \quad e10 = f_{fl}; \\
 e11 &= e7 - x[7]; \quad e12 = e8 - x[8]; \quad e13 = e9 - x[9]; \quad e14 = e10 - x[10]; \quad e15 = step(e11); \\
 e16 &= step(e12); \quad e17 = step(e13); \quad e18 = step(e14)
 \end{aligned} \tag{14}$$

It should be noticed that variables  $e7$ - $e18$  represent control variables for forces in rear-right, rear-left, front-right and front-left shock absorber ( $f_{arr}$ ,  $f_{arl}$ ,  $f_{afr}$ ,  $f_{afl}$ ), respectively – see fig. 2. Bearing in mind the adopted concept of damping control in the form of switch [44], in this paper forces in shock absorbers are defined as follows:

$$\begin{aligned}
 F_{arr} &= e15 \cdot A_{ar} \cdot (\dot{\Delta}_{rr} - \dot{z}_{rr}); \quad F_{arl} = e16 \cdot A_{ar} \cdot (\dot{\Delta}_{rl} - \dot{z}_{rl}) \\
 F_{afr} &= e17 \cdot A_{af} \cdot (\dot{\Delta}_{fr} - \dot{z}_{fr}); \quad F_{afl} = e18 \cdot A_{af} \cdot (\dot{\Delta}_{fl} - \dot{z}_{fl})
 \end{aligned} \tag{15}$$

where  $A_{af}$  and  $A_{ar}$  refer to maximal apertures the oil is flowing through, for front and rear shock absorbers.

Bearing in mind the remarks concerning the use of switch “on”-“off” [25], the relation for control of damping coefficient is adopted in the following form:

$$k = \begin{cases} k_{\max} & \forall \dot{z}_s (\dot{z}_s - \dot{z}_{w,s}) > 0 \\ 0 & \forall \dot{z}_s (\dot{z}_s - \dot{z}_{w,s}) \leq 0 \end{cases} \tag{16}$$

$s = fl, fr, rl, rr$

On the basis of fig. 2, it is obvious that variables  $e15$ - $e18$  have values 0 or 1, depending on the corresponding result of eq. (16).

Non-linear forces in springs, not being a subject to control, are defined by equations [3, 9-16, 23, 24, 33, 41, 42]:

$$\begin{aligned}
 F_{orr} &= x[11] \cdot (\Delta_{rr} - z_{rr}) + x[12] \cdot (\Delta_{rr} - z_{rr})^3; \quad F_{orl} = x[11] \cdot (\Delta_{rl} - z_{rl}) + x[12] \cdot (\Delta_{rl} - z_{rl})^3; \\
 F_{ofr} &= x[13] \cdot (\Delta_{fr} - z_{fr}) + x[14] \cdot (\Delta_{fr} - z_{fr})^3; \quad F_{ofl} = x[13] \cdot (\Delta_{fl} - z_{fl}) + x[14] \cdot (\Delta_{fl} - z_{fl})^3
 \end{aligned} \tag{17}$$

Radial forces in tires can be expressed on the basis of their radial deformations [13]:

$$F_{w,s} = c_{p1,s} \Delta_s + c_{p2,s} \Delta_s^2 - c_{p2,s} \Delta_s^3; \quad s = fl, fr, rl, rr \tag{18}$$

where tire deformations are given by equations:

$$\Delta_s = z_s - z_{o,s}; \quad s = fl, fr, rl, rr \tag{19}$$

Dynamic processes are described by eqs. (2-19). Bearing in mind their character and stochastic or shock character of road microroughness excitation, it is obvious that the equations should be solved numerically, by use of Runge-Kutta method. Numerical integration is carried out with time increment of 0.001 s, in 4096 points, which leads to reliable results in the frequency range 0.048-50 Hz [44-46], what was found acceptable from the aspect of vibration comfort and handling [20, 36].

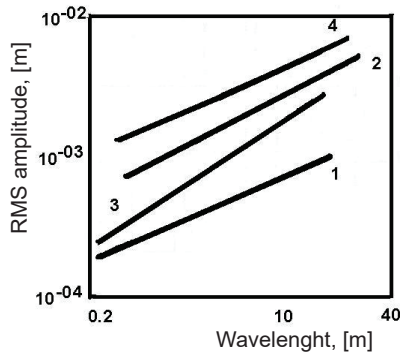
Referring to fig. 1, it is obvious that the observed vehicle model vibrates due to the excitation generated by the road micro-roughness, acting on front and rear wheels.

In the available literature there are many data on road microroughness [14, 37], most of them are based on definition of power spectra, what does not enable the inverse Fourier transform for determination of time function of excitation [14]. Therefore, a procedure for definition of polyharmonic time function of excitation from [14] will be adopted in this paper, and briefly described. Initial data on road microprofile are adopted from [14].

The excitation function (in time domain) coming from the road microroughness applied on the corresponding wheel is assumed to be in the following form:

$$z_{oi}(t) = \sum_1^{NH} A(f) \sin[2\pi ft + \varepsilon(f)] \quad (20)$$

where  $z_{oi}$ ,  $i - fl, fr, rl, rr$  – excitation functions coming from the road microroughness, acting on the corresponding wheel,  $A(f)$  – amplitude of road micro-roughness (in time domain),  $f$  – frequency,  $\varepsilon(f)$  – phase angle,  $t$  – time,  $NH$  – desired number of harmonics.



**Figure 3. Limit values of road roughness** (1 – modern road, 2 – road with a bad concrete surface, 3 – repaired asphalt road, and 4 – city street in good condition)

This study utilizes excitation function of 100 harmonics, which enables excitation distributed within the frequency interval approximately 0.015-35 Hz. That complies with vibration comfort requirements [38].

Having in mind the form of spatial distribution of amplitudes of road microroughness (fig.3) and taking into account vehicle velocity, the following equation can be written:

$$A(f)_{oi} = (A_0 + B_0 f_p) / v \quad (21)$$

where  $A_0$  and  $B_0$  are the coefficients dependent on the type of road surface,  $\lambda$  is the wavelength,  $f_p$  – the road unevenness wavelength ( $f_p = 1/\lambda$ ), and  $v$  – the vehicle velocity.

Lacking better data, the phase angle was defined by the following equations [15]:

$$\varepsilon(f)_{oi} = 2\pi(RND - 0.5); \quad i - fl, fr, rl, rr \quad (22)$$

where  $RND$  refers to a set of random numbers, uniformly distributed within the interval  $\{0, 1\}$ .

The same procedure and expressions (14-17) enable calculation of the excitation at the right side of the vehicle, due to the stochastic character of the expression (16), and thus provide different road excitations applied to the left and the right side of the vehicle.

The excitation function for rear-left wheel is defined as follows:

$$z_{ori} = z_{ori}(t - L/v) \quad (23)$$

where  $L = a+b$  – wheelbase.

Since the character of the relations (20)-(22) is stochastic, for the sake of simplification, the excitation on the right vehicle side can be described by relations (20)-(23).

For the illustration, fig. 4 shows a time function of the front left wheel. It is obvious on the fig. that the character of the applied excitation functions is clearly stochastic. The excitation functions for the other wheels can be calculated in the similar way, and will not be presented here.



Parameters of the observed vehicle are given in tab. 1.

It should be pointed out that the procedure can take into account the effect of variation of shock absorbers characteristics on its thermal characteristics, due to aging of shock absorber.

Damping parameters are being changed during the service life. To be more precise, values of forces in the shock absorber decrease.

For the illustration, fig. 5 shows a typical example of the mentioned variation [38], where it is assumed that values of the parameters decrease linearly with the duration of service, expressed in kilometers. Data in fig. 5 enable the analysis of the impact of variation of vibration parameters on dynamic characteristics of the vehicle.

A linear dependence has been adopted for the analysis of the influence of vehicle dynamic characteristics variation caused by variation of damping parameters due to aging. More precisely, by taking into account fig. 5, after a straight line approximation, any variation is defined by the equation:

$$k = 100 - 8.5 \cdot 10^{-5} S, \quad [\%] \quad (24)$$

where  $S$  is the current mileage [km], and  $k$  – the value of damping force degradation factor after operation of  $S$  kilometers.

The force of the shock absorber is now given by the equation:

$$F_{sh} = F_{sh0} \frac{k}{100} \quad (25)$$

where  $F_{sh}$  is the force in the damper after the operation of the vehicle for  $S$  kilometers, and  $F_{sh0}$  – the damper force at the beginning of exploitation.

Thermal loads of shock absorbers have been studied in this paper for the beginning of service life and after mileage of 200.000 km, using eqs. (24) and (25).

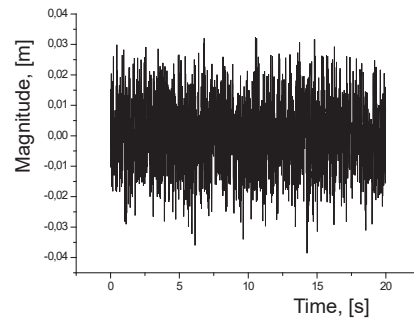


Figure 4. Excitation function of the road at the front left wheel

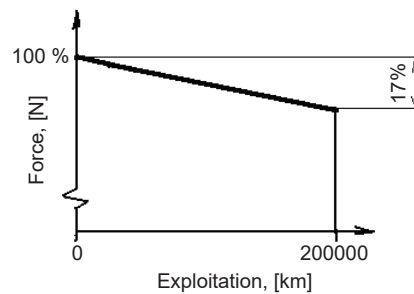


Figure 5. Variation of shock absorbers characteristic in service

Table 1. Basic data on the observed vehicle parameters

Sprung mass, $m_s$ , [kg]	1500**
Front wheel mass, $m_{wf}$ , [kg]	59
Rear wheel mass, $m_{wr}$ , [kg]	59
Moment of inertia, $I_x$ , [kgm <sup>2</sup> ]	750**
Moment of inertia, $I_y$ , [kgm <sup>2</sup> ]	2160**
Moment of inertia, $I_z$ , [kgm <sup>2</sup> ]	2160**
Wheelbase, $L$ , [m]	3.1
C.G. position, $a/b$ , [-]	1.4/1.7
* Linear coefficient of tire stiffness, $c_{l1}$ , [N/m]	190000
* Square coefficient of tire stiffness, $c_{l2}$ , [Nm <sup>-2</sup> ]	1900000
* Cube coefficient of tire stiffness, $c_{l3}$ , [Nm <sup>-3</sup> ]	19000000
Desired vehicle height, $z_{des}$ , [m]	0.5
Sprung mass desired roll, $\phi_{des}$ , [rad]	0
Sprung mass desired pitch, $\theta_{des}$ , [rad]	0
Characteristic vehicle velocity, $v$ , [ms <sup>-1</sup> ]	30

\* For front and rear tires

\*\* Inertial parameters for the unloaded vehicle are multiplied by factor 0.75

### Thermal load of shock absorber of a semi-active suspension system

Due to the relative motion of sprung and unsprung masses, the mechanical work is being done in shock absorbers, which is equivalent to the amount of heat  $Q$  [4-7, 39]. Mechanical work (the amount of heat) is defined by the expression [4, 7]:

$$A_i = \int_0^{s_r} F_{sh,i} ds_{r,i} = \int_0^T F_{sh,i} v_{r,i} dt; \quad i - fl, fr, rl, rr \quad (26)$$

where  $F_{sh,i}$  is the force in the shock absorber,  $s_{r,i}$  – the relative travel of the piston, with regard to the shock absorbers body (eq. 7),  $v_{r,i}$  – the relative velocity of the piston with regard to the shock absorbers body (time derivatives of displacements given in eq. 6), and  $t$  – the time.

Power required for causing relative movement of the piston, relative to the shock absorbers body, is given by the equation:

$$P(t)_i = F_{sh,i} v_{r,i}; \quad i - fl, fr, rl, rr \quad (27)$$

As is it well known from [5-9, 11, 40, 41], this power is identical to the heat flow (flux) for the heat transferred by convection, and its mean value is:

$$P_{mv,i} = \frac{1}{T} \int_0^T P(t)_i dt; \quad i - fl, fr, rl, rr \quad (28)$$

Dissipation of the most of the amount of heat to the surrounding air is by convection [4]. Formal dependence is given by the following equation [5]:

$$\bar{P}_i = \alpha_i S_i \Delta t; \quad i - fl, fr, rl, rr \quad (29)$$

where  $\alpha$  is the heat transfer coefficient,  $S$  – the area wherefrom the heat dissipates, and  $\Delta t$  – the temperature difference between the outside air and shock absorber,

As already noted, in this work the analysis of heat transfer from the absorber has not been carried out, because the values of  $\alpha_i$  and  $S_i$  are not known, and it requires very extensive experimental studies to determine these values, which will certainly be the subject of special attention in the future.

### Method and data analysis

For the sake of analysis of thermal loads of shock absorbers, a software for numerical solution of differential equations of vehicle motion has been developed in Pascal (eqs. 26 and 28). This will not be discussed due to space limitation. The algorithm is given in [17].

Specifically, the power (corresponding to the heat flux) and mechanical work (equivalent to the amount of heat) are calculated for each shock absorber. The analysis included velocities (30 and 5 m/s), degree of vehicle load (100%, 75%), and degradation of parameters of shock absorbers due to aging. The simulation relied on the eqs. (1)-(28).

For the illustration, heat flux of the front left shock absorber for the vehicle velocity of 30 m/s and fully loaded vehicle is shown in fig. 6. By analyzing the data of heat flux of all shock absorbers, an illustrative example is shown in fig. 6, it can be noted that thermal processes in shock absorbers of a semi-active suspension system, of a vehicle in motion on the road with stochastic parameters of microprofile, are also stochastic processes. Besides, the vehicle ve-

locity and vehicle load, as well as degradation of parameters of shock absorbers due to aging, affect the heat flux. Furthermore, there is the influence of the shock absorbers position relative to the vehicle C.G., which is expected, bearing in mind the vehicle and its systems vibration processes, see eq. (7).

It is of relevance to determine whether there is a connection between the excitation of road microprofile and heat flux. For this purpose, the ordinary coherence function is calculated for all four shock absorbers, using the program DEM-PARKOH [43].

The analysis of data on all coherence functions, for which an illustrative example is shown in fig. 7, it was found that it depends on the frequency, shock absorbers position, velocity and vehicle load, and total duration of service time. Value of the coherence function is within the interval, approximately 0.4-0.62, which indicates that the coupling between the excitation functions coming from the road exists. However, the fact that coherence is less than one, indicates that transformation of the energy from the road into heat flux of shock absorbers is expressed with certain non-linearities [44, 45]. This is understandable, having in mind that all elasto-damping elements have the impact on the transfer of energy, and that the mentioned energy transformation in the shock absorber, as a part of semi-active suspension system, is very complex.

Since heat flux is a stochastic value, it was found appropriate to calculate average power-fluxes [44-46] for each shock absorber separately, and the data are given in tab. 2. The analysis of data from tab. 2 can lead to the conclusion that heat fluxes of all shock absorbers decrease with the increase of vehicle velocity, which is logical considering that the excitation functions of road micro-profile decrease at higher velocities (eq. 21). Also, from tab. 2 it can be noticed that heat fluxes decrease with decreasing of vehicle load (inertial parameters). These

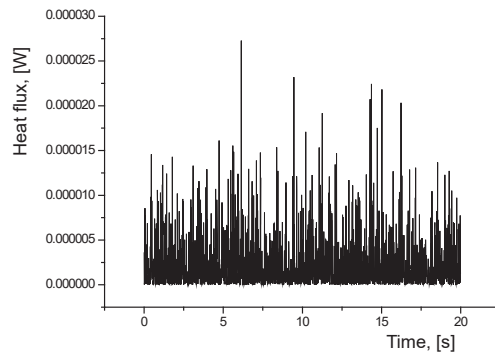


Figure 6. Heat flux for the velocity of 30 m/s, fully loaded vehicle and new shock absorbers

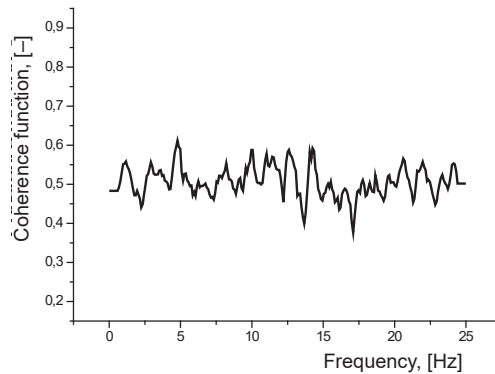


Figure 7. Ordinary coherence function “excitation-heat flux” for the front left wheel, and partially loaded vehicle for a speed of 5 m/s and new shock absorbers

Table 2. Mean heat flux of shock absorbers (new / 200.000 km service)

Mean heat flux, [W]	Front left shock absorber	Front right shock absorber	Rear left shock absorber	Rear right shock absorber
Velocity 30 m/s Mass 100%	2.330 exp(-006) / 1.934 exp(-006)	5.569 exp(-006) / 4.622 exp(-006)	5.358 exp(-006) / 4.447 exp(-006)	2.944 exp(-006) / 2.443 exp(-006)
Velocity 5 m/s Mass 100%	2.435 exp(-006) / 2.021 exp(-006)	5.673 exp(-006) / 4.709 exp(-006)	5.477 exp(-006) / 4.546 exp(-006)	2.968 exp(-006) / 2.464 exp(-006)
Velocity 5 m/s Mass 75%	2.431 exp(-006) / 2.011 exp(-006)	4.518 exp(-006) / 3.866 exp(-006)	4.263 exp(-006) / 3.608 exp(-006)	2.748 exp(-006) / 2.388 exp(-006)

findings apply to both new and shock absorbers that are already in use for 200,000 km. In addition, from tab. 2 it can be noticed that thermal loads of shock absorbers that were already in use (200,000 km) are less than with the new ones. This is explained by the fact that the force of a used shock absorber is smaller for about 17%, and the same thing is happening with the power in them that is converted into heat.

It is of relevance to analyse the mechanical work, which is turned into heat in shock absorbers, for a certain travel distance. As this work is growing with time, it was assumed appropriate to calculate it for the duration of 20 s. The calculation is performed both for partly or totally loaded vehicle, for two cases of vehicle velocity, 5 and 30 m/s, and for new and shock absorbers being in service for 200,000 km.

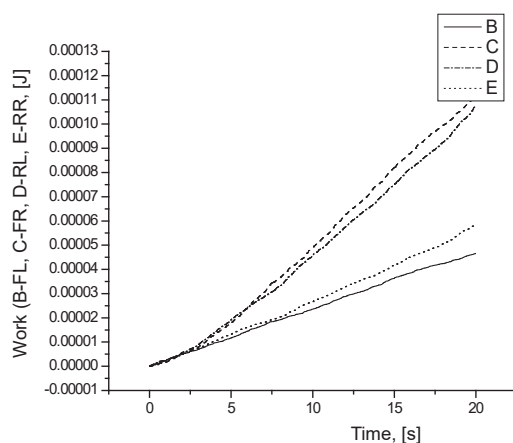
For the illustration, fig. 8 shows the work that is converted into heat in shock absorbers of a semi-active suspension system, for the velocity of 5 m/s and a fully loaded vehicle. In figs 8-10, B – refers to the Front Left shock absorber, C – refers to the Front Right shock absorber, D – refers to the Rear Left shock absorber, and E – refers to the Rear Right shock absorber.

The analysis of data on the amount of heat for the velocity of 5 and 30 m/s (an illustrative example for the velocity of 5 m/s is given in fig. 8), can lead to the conclusion that the production of the amount of heat increases with the travel (duration time) and is higher for the lower velocities of the vehicle, which complies with the interpretation of the heat flux, the lower velocities whose differential it is.

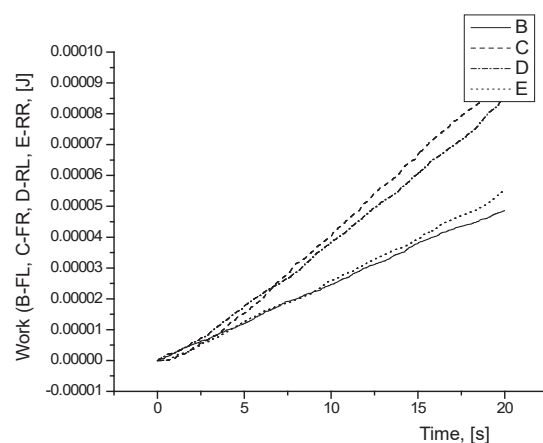
To determine the effect of vehicle load on the amount of heat generated in shock absorbers of a semi-active suspension system of the vehicle, the illustrative example from the fig. 9 will be observed. The figure shows the mechanical work (converted into heat) which is done in shock absorbers of a partially loaded vehicle (75%) while moving with the velocity of 5 m/s.

Based on the data from figs. 8 and 9 it can be determined that the reduced load of the vehicle leads to the less thermal load of vehicle shock absorbers, as well as heat flux.

To determine the effects of degradation of shock absorbers characteristics due to aging, fig. 10, as an example, shows the results of the mechanical work in shock absorbers of a fully loaded vehicle, with the velocity of 5 m/s and shock absorbers already being in service for 200,000 km.



**Figure 8.** Mechanical work (amount of heat) in shock absorbers for the velocity of 5 m/s of a fully loaded vehicle with new shock absorbers



**Figure 9.** Mechanical work (amount of heat) in shock absorbers for the velocity of 5 m/s of a partly loaded vehicle with new shock absorbers

The analysis of all data, for which an illustrative example is given in fig. 10, enabled the notice that mechanical work decreases with the duration of service time, which is also manifested for the heat flux, what was already discussed above.

### Conclusions

Based on the performed research, the following can be concluded.

- Thermal load of shock absorbers can be calculated by use of a developed vibration model of vehicle with semi-active suspension system and software in Pascal.
- Production of heat in shock absorbers decreases with the increase of vehicle velocity, during its motion along the road of identical properties.
- Production of heat in shock absorbers increases with the increase of the vehicle load.
- Thermal load of shock absorbers decrease with the increase of service time duration.
- Study of thermal load of shock absorbers gives necessary elements for shock absorber design so as to prevent excessive heat that might damage sealing elements and consequently lead to a variation of the designed damping characteristics.

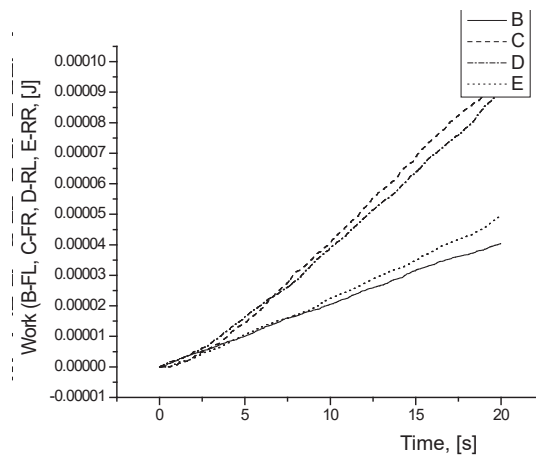


Figure 10. Mechanical work (amount of heat) in shock absorbers for the velocity of 5 m/s of a fully loaded vehicle with shock absorbers already in service for 200,000 km

### Nomenclature

$A$  – work, [J]  
 $e$  – control variables for forces [N]  
 $F_{sh}$  – force in the shock absorber, [N]  
 $f$  – frequency, [Hz],  
 $I$  – moment of inertia, [kgm<sup>2</sup>]  
 $k$  – value of damping force degradation factor, [-]  
 $NH$  – desired number of harmonics, [-]  
 $Q$  – quantity of heat, [J]  
 $S$  – current mileage, [km]  
 $t$  – time, [s],  
 $z$  – vertical vibration, [m]

### Greek symbols

$\varepsilon(f)$  – phase angle, [rad]  
 $\theta$  – pitch, [rad]  
 $\lambda$  – wavelength, [m]  
 $\varphi$  – roll, [rad]

### Subscripts

$fl$  – front left  
 $fr$  – front right  
 $rl$  – rear left  
 $rr$  – rear right  
 $x$  – reference for x-axis  
 $y$  – reference for y-axis  
 $z$  – reference for z-axis

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