CALCULATIONAL AND EXPERIMENTAL INVESTIGATIONS OF VOID EFFECT A NEW MODEL FOR LEAKAGE TREATMENT OF HETEROGENEOUS ASSEMBLIES

P. BENOIST*, J. MONDOT**, I. PETROVIC***

Commissariat à l'Energie Atomique
Direction des Réacteurs Nucléaires
* Département de Mécanique et de Technologie
*** Service d'Etudes de Réacteurs et de Mathématiques Appliquées
Centre d'Etudes de Saclay
91191 Gif sur Yvette Cedex - FRANCE

** Commissariat à l'Energie Atomique
Direction des Réacteurs Nucléaires
Département d'Etudes des Réacteurs
Service de Physique des Réacteurs et du Cycle
Centre d'Etudes Nucléaires de Cadarache
13108 Saint Paul lez Durance Cedex - FRANCE

ABSTRACT

The growing interest in safety problems has led physicists to try to increase their knowledge of the neutron leakage phenomenon, both by calculation and by experiment. The flux calculation in a heterogeneous assembly is frequently performed by collision probability method. On the contrary, neutron leakage is calculated for a flux-weighted homogenized assembly, which is a good approximation for a non-voided assembly (PWR in normal conditions). In a LOCA situation, the assembly may contain void zones, and this model for leakage calculation may became insufficient. We propose here a new theoretical model, taking into account the effect of heterogeneity of the assembly on neutron leakage, and which was implemented as TIBERE procedure in the multigroup transport assembly code APOLLO-2. One of the advantages of this new model is to allow a perfectly consistent definition of cell reaction and cell leakage rates used in the equivalence procedure. As well as this theoretical work was made, an experimental program concerning this phenomenon was performed as a part of EPICURE experiment. Comparisons of experimental and calculational results point out better agreement of the new model with the measurements.

INTRODUCTION

The increasing interest in safety requirements of nuclear reactors had led physicists to devote more attention to certain problems which could reasonably be considered of secondary importance in normal operation situations, at least in the case of PWRs. In particular, the effect of lattice heterogeneity on neutron leakages seems to have been somewhat neglected in PWR assembly calculations; indeed, a very fine description of the multigroup flux chart can be obtained in the present, but leakages are calculated for a flux-weighted homogenized assembly, which is a rough representation of actual leakages. This approximation can lead to an important underestimation of leakages in LOCA situations, where voids or fogs appear in an assembly. Therefore, a work has been undertaken at the CEA in order to improve the knowledge of this effect: an experimental programme EPICURE[1], was performed at Cadarache, a part of which was devoted to the void effect; at the same time the elaboration of a new model for leakage calculation[2], TIBERE, was carried out at Saclay, and implemented in the assembly

code[3] APOLLO-2. The comparison between experiment and calculation is given in this paper. This new model allows to take into account the effect of the heterogeneity of the assembly on leakages, which was not possible in the previous procedure.

THE APOLLO-2 CODE

The APOLLO-2 code (as well as APOLLO[4]) first calculates the fine multigroup flux chart in the volume V_A of a heterogeneous perfectly reflected assembly (equivalent to an infinite lattice of assemblies), for instance by the "2D-xy exact" collision probability module MARSYAS[5]. The flux is the solution of the multigroup integral transport equation :

$$\varphi^g(\vec{r}) = \int_{V_A} d^3r' \frac{\exp[-\tau^g(\vec{r},\vec{r}')]}{4\pi R^2} \sum_{g'} \hat{\Sigma}_o^{g'-g} \varphi^{g'}(\vec{r}')$$

with

$$R = |\vec{r} - \vec{r}'| \qquad \hat{\Sigma}_o^{g-g}(\vec{r}') = \Sigma_{so}^{g'-g}(\vec{r}') + \chi^g \frac{\nu\Sigma_f^{g'}(\vec{r}')}{k_\infty}$$

After discretization in calculational zones $j \in V_A$, this equation becomes, φ_j^g being the average flux in zone j (total cross section Σ_j^g) :

$$V_j \Sigma_j^g \varphi_j^g = \sum_{i \in V_A} V_i P_{ij}^g \sum_{g'} \hat{\Sigma}_{oi}^{g'-g} \varphi_i^{g'}$$

The P_{ij}^g are the first flight collision probabilities from zone i to zone j in group g. Once obtained the multigroup flux chart φ_j^g, one may define, by flux weighting, a homogenized medium "equivalent" to the assembly; for any reaction β :

$$\bar{\Sigma}_{\beta j}^g = \frac{\langle \Sigma_\beta^g \varphi^g \rangle_A}{\langle \varphi^g \rangle_A} \qquad \langle \ \rangle_A = \int_{V_A} \ldots\ldots d^3r$$

This medium, homogeneous and perfectly reflected (or homogeneous and infinite) is not critical; in order to make it critical, macroscopic leakages are to be introduced, in such a way that the angular flux is :

$$\Psi^g(\vec{r},\vec{\Omega}) = \psi^g(\vec{\Omega}) \exp(i\vec{B}\cdot\vec{r})$$

where $\vec{B}$ is the bucking vector. Introducing this flux in the homogeneous and infinite transport equation, the critical buckling B_o^2 and the leakage coefficient D^g of the medium are determined, using the linearly anisotropic scattering approximation (homogeneous B_1 procedure). This critical buckling is not the fundamental buckling of the actual heterogeneous assembly. Therefore a second heterogeneous calculation is performed, introducing leakages as an additional absorption cross-section $D^g B_o^2$. A new flux chart is obtained, and the procedure is continued by iterations until convergence is attained. The drawback of this procedure comes from the fact that leakages are calculated for a homogenized assembly, not for the actual assembly; the leakage cross-section should indeed depend on point $\vec{r}$. which is not the case here.

In an actual reactor, the assembly is not perfectly reflected. The full calculation of a reactor being, at least at the present, almost impossible, an equivalence procedure is necessary. In each assembly, a heterogeneous macroregion M, for instance a cell, is replaced by an equivalent homogeneous medium; moreover, instead of treating the problem in the fine group g representation, the cross-sections are condensed into a small number of macrogroups G. These cell-condensed cross-sections, used in the *core calculation, will be determined in each assembly by a transport-diffusion (or transport-transport)* equivalence procedure. This procedure requires the knowledge of the reaction rate $T^G_{M,\beta}$ in the macroregion M and in the macrogroup G; for the reaction β :

$$T^G_{M,\beta} = < \Sigma^g_\beta \varphi^g >^G_M \qquad < >^G_M = \sum_{g\in G} \int_{V_M} .....d^3r \tag{1}$$

But the leakage rates $T^G_{A,l}$ are defined, not at the scale of a macroregion M, but at the scale of the whole assembly A :

$$T^G_{A,l} = < D^g B_o^2 \varphi^g >^G_A$$

Using reaction and leakage rates, average cross-sections and leakage coefficient can be defined

$$\bar{\Sigma}^G_{M,\beta} = \frac{T^G_{M,\beta}}{< \varphi >^G_M} \qquad \bar{D}^G_A = \frac{T^G_{A,l}}{B_o^2 < \varphi >^G_A}$$

It has been shown[6,7] that this simple weighting does not satisfie the equivalence of reaction rates and leakage rate between the "reference" transport calculation and the "macrocalculation". The equivalence is only possible for certain homogenized parameters defined as :

$$\tilde{\Sigma}^G_{M,\beta} = \mu^G \bar{\Sigma}^G_{M,\beta} \qquad \tilde{D}^G_A = \mu^G \bar{D}^G_A$$

where μ^G, the SPH factor, is obtained by a non-linear procedure. This procedure contains certain inconsistency due to the fact that the equivalent cross-sections $\tilde{\Sigma}^G_M$ are defined at the scale of the *macroregion* M, while the leakage coefficient D^G_A is defined only at the scale of the assembly A.

HETEROGENEOUS B_1 FORMALISM

The transport equation for the angular flux $\Psi^g(\vec{r}, \vec{\Omega})$ is :

$$H\Psi = P\Psi$$

with, Σ^g being the total cross section (diagonal multigroup matrix):

$$H\Psi = \vec{\Omega} \cdot \vec{\nabla} \Psi^g + \Sigma^g \Psi^g \tag{2}$$

and, in the linearly anisotropic scattering approximation :

$$P\Psi = \sum_{g'} [\hat{\Sigma}_o^{g'-g}(\vec{r}) \Phi^{g'}(\vec{r}) + \Sigma_1^{g'-g}(\vec{r}) \sum_k \Omega_k J_k^{g'}(\vec{r})]$$

where, Ω_k being the k-component of the unit vector $\vec{\Omega}$:

$$\Phi^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\Psi^g(\vec{r},\vec{\Omega})\,d\Omega \qquad J_k^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\Psi^g(\vec{r},\vec{\Omega})\,\Omega_k\,d\Omega$$

The heterogeneous B_1 formalism (which is itself an approximation, since in an assembly, surrounded by assemblies of different types, flux transients will appear near the boundary) lies on the following hypothesis[8, 9] :

$$\Psi^g(\vec{r},\vec{\Omega}) = \exp(i\vec{B}\cdot\vec{r})\,\psi^g(\vec{r},\vec{\Omega})$$

where $\vec{B}$ is the buckling vector. Substituting this expression in Eq.2 we obtain

$$(H + i\vec{B}\cdot\vec{\Omega})\,\psi = P\psi \qquad (3)$$

In an infinite medium of identical assemblies, $\psi^g(\vec{r},\vec{\Omega})$ has the periodicity of the assembly (assumed to be symmetrical). This angular flux ψ^g is complex[8] :

$$\psi^g = \psi_s^g - \sum_k B_k\,\psi_{ak}^g \qquad (k = x, y, z)$$

where $\psi_s^g(\vec{r},\vec{\Omega})$ and $\psi_{ak}^g(\vec{r},\vec{\Omega})$ depend on vector $\vec{B}$. At the scale of the assembly, the angle-integrated flux $\varphi_s^g(\vec{r})$ corresponding to $\psi_s^g(\vec{r},\vec{\Omega})$ is symmetrical with respect to direction k, while the angle-integrated flux $\varphi_{ak}^g(\vec{r})$ corresponding to $\psi_{ak}^g(\vec{r},\vec{\Omega})$ is anti-symmetrical with respect to direction k and symmetrical with respect to directions $k' \neq k$ (If the cross-sections are independent of the axial directions k = z (2D - problem), φ_s^g, φ_{ax}^g and φ_{ay}^g are independent of z and φ_{az} is identically zero). The angular fluxes ψ_s^g and ψ_{ak}^g may be chosen as solutions of the system :

$$\begin{aligned} H\psi_s &= \hat{\Sigma}_o\,\varphi_s + \Sigma_1\sum_k \Omega_k\,j_{sk} - \sum_k\sum_{k'} B_k\,B_{k'}\,\Omega_k\,\psi_{ak'} \\ H\psi_{ak} &= \hat{\Sigma}_o\,\varphi_{ak} + \Sigma_1\sum_{k'}\Omega_k\,j_{akk'} + \Omega_k\,\psi_s \end{aligned} \qquad (4)$$

with

$$\varphi_s^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\psi_s^g(\vec{r},\vec{\Omega})\,d\Omega \qquad j_{sk}^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\psi_s^g(\vec{r},\vec{\Omega})\,\Omega_k\,d\Omega$$

$$\varphi_{ak}^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\psi_{ak}^g(\vec{r},\vec{\Omega})\,d\Omega \qquad j_{akk'}^g(\vec{r}) = \frac{1}{4\pi}\int_{4\pi}\psi_{ak}^g(\vec{r},\vec{\Omega})\,\Omega_{k'}\,d\Omega$$

The symbols $\hat{\Sigma}_o$ and Σ_1 are multigroup transfer matrices, and the fluxes and currents are multigroup vectors. The solution of such a heterogeneous B_1 system would be possible and was already carried out in some particular geometries[10]. But it would be certainly very time-consuming in complex assemblies. Moreover we must remind that the heterogeneous B_1 procedure is itself an approximation.

A SIMPLIFIED HETEROGENEOUS B_1 PROCEDURE: TIBERE

Therefore we propose a simplified form of the heterogeneous B_1 procedure which, due to some approximations, requires only mathematical tools existing in APOLLO-2, or very similar tools. Antisymmetry considerations, rigorous for k = z, approximate for k = x, y, allow to cancel j_{sk} from the first equation of Eq.4 , φ_{ak} from the second one, as well as cross-terms k' ≠ k. This approximation, consisting in neglecting the influence of antisymmetrical terms, corresponds to neglecting the angular correlation terms. These terms, neglected in the first streaming theories[11], have been shown[12,13] to be very important in large cells such as in graphite or heavy water reactors, but can be reasonably considered as negligible[14] in assemblies such as for PWR. Equation 4 becomes then :

$$\begin{aligned} H\psi_s &= \hat{\Sigma}_o \varphi_s - \sum_k B_k^2 \Omega_k \psi_{ak} \\ H\psi_{ak} &= \Sigma_1 \Omega_k j_{akk} + \Omega_k \psi_s \end{aligned} \qquad (5)$$

Moreover let us admit for the moment that, on the RHS of the second Equation 5 :

$$\psi_s^g(\vec{r},\vec{\Omega}) \# \varphi_s^g(\vec{r}) \qquad \text{(isotropic approximation)}$$

and that, on the RHS of the first Equation 5 :

$$\psi_{ak}^g(\vec{r},\vec{\Omega}) \# 3\, j_{akk}^g(\vec{r})\, \Omega_k \qquad \text{(anti-isotropic approximation)}$$

Equation 5 becomes now :

$$\begin{cases} H\psi_s = \hat{\Sigma}_o \varphi_s - 3 \sum_k B_k^2 \Omega_k^2 j_{akk} \\ H\psi_{ak} = \Sigma_1 \Omega_k j_{akk} + \Omega_k \psi_s \end{cases} \qquad (6)$$

In the particular case of a homogeneous medium H, Equation 6 reduces (in multigroup matrix notation of course), to :

$$\begin{cases} \Sigma^H \varphi_s^H = \hat{\Sigma}_o^H \varphi_s^H - B^2 j_a^H & \text{(conservation relation)} \\ \Sigma^H j_a^H = \frac{1}{3}(\Sigma_1^H j_a^H + \varphi_s^H) & \text{(Fick's relation)} \end{cases}$$

The second relation is only approximate; the exact relation (given by the homogeneous B_1 calculation) would be :

$$\Sigma^H j_a^H = \frac{1}{3} \Lambda \left(\frac{B}{\Sigma^H}\right) (\Sigma_1^H j_a^H + \varphi_s^H)$$

where, Σ^{Hg} depending of course on the group g :

$$\Lambda\left(\frac{B}{\Sigma^H}\right) = \frac{3\,\Sigma^H}{B}\,\frac{1-\frac{\Sigma^H}{B}\arctan^{-1}\frac{B}{\Sigma^H}}{\arctan^{-1}\frac{B}{\Sigma^H}} \qquad \left(\Lambda\left(\frac{B}{\Sigma^H}\right) \text{ is very near to one}\right)$$

We propose to perform the same operation on the second heterogeneous Equation 6 by the approximate correction :

$$H\psi_{ak} = \Lambda\left(\frac{B}{\Sigma^H}\right)\left(\Sigma_1\,\Omega_k\,j_{akk} + \Omega_k\,\varphi_s\right) \tag{7}$$

where Σ^H is obtained by simple flux-weighting, and where $B^2 = \sum_k B_k^2$. The system of Equation 6 (the second equation being replaced by Eq.7) may be written in the integral form :

$$\begin{cases} \varphi_s^g(\vec{r}) = \displaystyle\int_{V_A} d^3r' \;\frac{\exp\left[-\tau^g(\vec{r},\vec{r}')\right]}{4\pi R^2}\left[\sum_{g'}\hat{\Sigma}_o^{g'-g}(\vec{r}')\,\varphi_s^{g'}(\vec{r}') - 3\sum_k B_k^2\,\Omega_k^2\,j_{akk}^g(\vec{r}')\right] \\ j_{akk}^g(\vec{r}) = \Lambda\left(\frac{B}{\Sigma^{Hg}}\right)\displaystyle\int_{V_A} d^3r' \;\frac{\exp\left[-\tau^g(\vec{r},\vec{r}')\right]}{4\pi R^2}\,\Omega_k^2\left[\sum_{g'}\Sigma_1^{g'-g}(\vec{r}')\,j_{akk}^{g'}(\vec{r}') + \varphi_s^g(\vec{r}')\right] \end{cases} \tag{8}$$

The discretization of Equation 8 gives :

$$\begin{cases} V_j\,\Sigma_j^g\,\varphi_{s,j}^g = \displaystyle\sum_{i\in A} V_i\left[P_{ij}^g\sum_{g'}\hat{\Sigma}_{o,i}^{g'-g}\,\varphi_{s,i}^{g'} - \sum_k B_k^2\,P_{ij,k}^g\,j_{akk,i}^g\right] \\ V_j\,\Sigma_j^g\,j_{akk,j}^g = \dfrac{1}{3}\,\Lambda\left(\dfrac{B}{\Sigma^{Hg}}\right)\displaystyle\sum_{i\in A} V_i\,P_{ij,k}\left[\sum_{g'}\Sigma_{1,i}^{g'-g}\,j_{akk,i}^{g'} + \varphi_{s,i}^g\right] \end{cases} \tag{9}$$

The P_{ij}^g are the classical first flight collision probabilities in group g, already used previously :

$$P_{ij}^g = \frac{\Sigma_j^g}{V_i}\int_{V_j} d^3r \int_{V_i} d^3r\;\frac{\exp\left[-\tau^g(\vec{r},\vec{r}')\right]}{4\pi R^2} \qquad R = |\vec{r}-\vec{r}'|$$

The $P_{ij,k}^g$ are the directional collision probabilities[12, 13] in direction k :

$$P_{ij,k}^g = \frac{\Sigma_j^g}{V_i}\int_{V_j} d^3r \int_{V_i} d^3r'\;\frac{\exp\left[-\tau^g(\vec{r},\vec{r}')\right]}{4\pi R^2}\cdot 3\,\Omega_k^2 \qquad \vec{\Omega} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

In a x-y problem, the P_{ij}^g and $P_{ij,k}^g$ are both integrals of Bickley functions. The multigroup flux and current charts are obtained, in each calculational zone, by solving the system Eq.9 by an iterative

scheme. In practice, instead of calculating separately j_{axx} and j_{ayy} TIBERE calculates their average value j_{arr} (radial).

Remark

The classical APOLLO-2 procedure presented previously is equivalent to solving, by an iteration scheme, the system of ***non-linear*** *equations :*

$$\varphi_s^g(\vec{r}) = \int_{V_A} d^3r' \; \frac{\exp[-\tau^g(\vec{r},\vec{r}')]}{4\pi R^2} \left[\sum_{g'} \hat{\Sigma}_o^{g'\to g}(\vec{r}')\,\varphi_s^{g'}(\vec{r}') - B^2 \frac{j_a^{Hg}}{\varphi_s^{Hg}} \varphi_s^g(\vec{r}) \right]$$

$$j_a^g = \Lambda\left(\frac{1}{\Sigma^{Hg}}\right) \frac{1}{3\Sigma^{Hg}} \left[\sum_{g'} \Sigma_1^{Hg'\to g}\, j_a^{Hg'} + \varphi_s^{Hg} \right] \qquad (10)$$

where the quantities with index H are calculated with the homogenized cross-sections :

$$\Sigma_\beta^{Hg} = \frac{\int_{V_A} \Sigma_\beta^g(\vec{r}')\,\varphi_s^g(\vec{r}')\,d^3r}{\int_{V_A} \varphi_s^g(\vec{r}')\,d^3r}$$

The ratio $j_a^{Hg} / \varphi_s^{Hg}$ appearing in the RHS of the first Eq.10 is the leakage coefficient D^g of the homogenized assembly. On the contrary, Eq.8 of the TIBERE model is ***linear****.*

The TIBERE model allows to establish a completely consistent equivalence procedure, treating the reaction "leakages" exactly as the other reactions (let us note that the idea of defining a cell leakage coefficient was proposed in Ref.15,17). If $\psi^g(\vec{r},\vec{\Omega})$ is the complex flux solution of Eq.3, a reference reaction rate can be defined for each reaction β, in the macroregion M and in the macrogroupe G :

$$T_{M,\beta}^G = [\Sigma_\beta \psi]_M^G = \langle \Sigma_\beta \varphi_s \rangle_M^G$$

with

$$[\;]_M^G = \sum_{g\in G} \int_{V_M} \int_{4\pi} \ldots\ldots d\Omega\, d^3r \qquad \langle\;\rangle_M^G = \sum_{g\in G} \int_{V_M} \ldots\ldots d^3r$$

The **antisymmetrical** part ψ_{ak} of ψ cancels identically for any "physical" reaction β. The expression of the reference reaction rate is the same as in the classical equivalence procedure (Eq.1). Let us define now reference reaction rates $T_{M,k}^G$ for the "volumetric leakage" reactions in direction k, in the macroregion M and in the macrogroupe G (this rate corresponds to the cross-section $iB_k\,\Omega_k$ appearing in Eq.3).

$$T_{M,k}^G = [iB_k\,\Omega_k \psi]_M^G = B_k^2 \langle j_{akk} \rangle_M^G$$

Here the contribution of the **symmetrical** part ψ_s of ψ cancels identically. The knowledge of reference rates $T_{M,\beta}^G$ and $T_{M,k}^G$ allows to define average cross-sections and leakage coefficients :

$$\bar{\Sigma}^{G}_{M,\beta} = \frac{T^{G}_{M,\beta}}{<\varphi_s>^{G}_{M}} \qquad \bar{\Sigma}^{G}_{M,k} = \bar{D}^{G}_{M,k} B^2_k = \frac{T^{G}_{M,k}}{<\varphi_s>^{G}_{M}}$$

But we have seen that this simple weighting did not satisfy the equivalence of reaction and leakage rates between the reference calculation and the "macrocalculation" (diffusion or transport calculation). The equivalence is only possible for the following homogenized cross-sections and leakage coefficients, where μ is the SPH factor :

$$\tilde{\Sigma}^{G}_{M,\beta} = \mu^{G} \bar{\Sigma}^{G}_{M,\beta} \qquad \tilde{D}^{G}_{M,k} = \mu^{G} \bar{D}^{G}_{M,k}$$

NUMERICAL CALCULATIONS

First we performed calculations of typical PWR, 17x17 cells, perfectly reflected assemblies. For the **non-voided** assembly the infinite multiplication factor is K_{inf} = 1.28347. The critical buckling obtained by the classical APOLLO-2 procedure is B^2 = 48.338 x 10^{-4}cm^{-2}. Assuring the axial buckling B^2_z = 16.000 x 10^{-4}cm^{-2}, TIBERE procedure gives the critical radial buckling B^2_r = 32.038 x 10^{-4}cm^{-2}, i.e. the total buckling B^2 = 48.038 x 10^{-4}cm^{-2}. A simple calculation showed that the classical method would overestimate the reactivity by approximately $\delta(\frac{1}{K})$ = 138 x 10^{-5}. Further on, from the central part (7 x 7 cells) of the same assembly, water was substituted by **void**. The infinite multiplication factor is now K_{inf} = 1.27061. The critical buckling obtained by the classical APOLLO-2 procedure is B^2 = 37.043 x 10^{-4} cm^{-2}. Assuming the same axial buckling as in the previous case (B^2_z = 16.000 x 10^{-4}cm^{-2}), the TIBERE procedure gives the critical radial buckling B^2_r = 19.469 x 10^{-4}cm^{-2}, i.e. the total buckling B^2 = 35.469 x 10^{-4}cm^{-2}. The classical method overestimates the reactivity by 935 x 10^{-5}. Perhaps more interesting than reactivities are the charts of the flux φ_s and of the currents j_{arr} (radial) and j_{azz} (axial) given by TIBERE in the fast and thermal groups. In the fast group (Fig. 1) the flux φ_s is practically uniform, but the currents have an important peak in the voided zone, in such a way that the leakage coefficients $D_r = \frac{j_{arr}}{\varphi_s}$ and $D_z = \frac{j_{azz}}{\varphi_s}$ have also an important peak in this zone; on the contrary the leakage coefficient resulting from the classical procedure would be uniform in the assembly, since in this procedure j_a is proportional to φ_s. In the thermal group (Fig. 2), the flux φ_s obtained by TIBERE is strongly depressed in the voided zone, due to the fact that there exists absorption in the fuel, but practically no thermal sources since there is no water; anyway the currents have still an important peak in this zone in such a way that the leakage coefficients $D_r = \frac{j_{arr}}{\varphi_s}$ and $D_z = \frac{j_{azz}}{\varphi_s}$ have also an important peak; hereagain the classical procedure would lead to a uniform leakage coefficient, since the ratio $\frac{j_a}{\varphi_s}$ is uniform.

COMPARISON BETWEEN EPICURE EXPERIMENTAL RESULTS AND CALCULATIONS

A part of the EPICURE experimental programme[1] is devoted to the investigation of two dimensional void effects both in UO2 and MOX lattices. The results of the experimental campaign in a UO2 regular lattice are available, while the measurements in the MOX cores are still to be done. The aim of these experiments is to measure the perturbation caused by the progressive removal of the water volume in a central zone of a regular core, in terms of reactivity and pinwise fission rate distributions, and to check the validity of our calculational schemes for such configurations. First, a reference core made of a regular lattice of UO2 (3.7 % 235U enriched) pins has been constructed. This reference core

has been carefully characterised using buckling measurements derived from a number of different reaction rate distributions, resulting in an uncertainty of 150 mN (2σ) on the corresponding Keff. At the center of this core, a 7 x 7 pins zone has been reserved for special geometrical changes. The three configurations devoted to the void effect correspond to the gradual removal of the water from this region by substituting with an equivalent volume of aluminium. The 30 % and 50 % void has been obtained simply, by increasing the thickness of the standard aluminium overclad, the purpose of which is to simulate the reduced water density under operating conditions (300 °C, 155b). The "100 % void" has been simulated and corresponds to the case where the 7 x 7 pins region is replaced by a solid block of aluminium containing 49 channels in which the UO2 pins are inserted. In each case, the following measurements have been made; criticality has been obtained using a compensating boric acid adjustment in the moderator, and pinwise fission rate distributions have been obtained from integral gamma scanning on pins themselves. The complete region comprising the perturbed zone and the surrounding region (extended until the asymptotic spectrum is recovered) has been investigated. The approximate shape of the fission rate distributions for the four cases (0, 30, 50 and 100 % of void) is illustrated in the Figure 3. One can observe that the depression of the fission rates in the central part increases with the void percentage, but that these perturbations vanish rather quickly and that the fundamental mode is recovered at a distance of approximately four pins from the voided-unvoided zone interface. The perturbation on both edges of the core is due to the presence of a water reflector.

All calculations performed concern the complete (100 %) replacement of water by aluminium in the voided zone. The radial buckling was determined by fitting on the experimental fundamental mode in the non-voided case. Indeed the experimental results show that there exists, not too near the boundaries of the reflector, the fundamental mode zone in which the spectrum index $\varphi_{fast}/\varphi_{thermal}$ does not depend on the space variable. This defines the flux extrapolated annulation point. In the voided case, one can observe, in the same way, the existence of a fundamental mode zone between the voided zone and the reflector. Hence, our hypothesis will be that the flux extrapolated annulation point is the same as in the non-voided case (in other terms, transients from voided zone and reflector do not overlap). The extrapolation-point was defined as R = 34.15 cm. In order to be able to calculate the reactor core by diffusion theory, we were obliged to define the square assembly (21 x 21 cells) containing the zone where water was replaced by aluminium, in such a way that its boundary lies inside the fundamental mode zone. In the reference calculation this assembly will be treated as perfectly reflected. Reference transport cell reaction rates and leakage coefficients are defined, and, by a SPH transport-diffusion equivalence, diffusion cross-sections and leakage coefficients are deduced. This calculation is made both by classical and by TIBERE leakage calculation procedures of APOLLO-2 code. It is assumed that, in the diffusion calculation, diffusion coefficients are equal to leakage coefficients. The whole core 2D diffusion calculation was carried out by the diffusion code CRONOS[16], assuming that the cells situated around the assembly were homogenized by simple flux-weighting. First we compared the reactivities of EPICURE in the voided case (aluminium), given by experiment, and by the classical and TIBERE procedures. The voided experiment is critical for a certain concentration of boron (383 p.p.m.). For the same concentration of boron, the reactivity obtained by CRONOS with the classical leakage calculation is -948 x 10^{-5} while CRONOS with TIBERE gives -1165 x 10^{-5} This decrease of reactivity (217.10^{-5}) is due to the fact that TIBERE increases the leakages compared to the classical procedure. The absolute error ($\sim -1000 \times 10^{-5}$) between the experiment and the calculations is due to the approximate modelling of the reactor, but this should not influence the difference between these two types of calculations. The most interesting comparison concerns the fission rate distribution in the EPICURE reactor. Here we present the relative discrepancy ($\frac{EXP - CALC}{EXP}$ %) between the

experiment and the classical leakage procedure (Fig. 4) and the TIBERE procedure (Fig. 5). The normalization of the calculated chart is chosen in such a way that the relative discrepancy is zero in the tenth cell of the diagonal, i.e. in the zone of the fundamental mode. We observe that the classical leakage model (Fig. 4) underestimates the depression in the voided zone (~ 3 %), while TIBERE gives a good agreement with experiment (the experimental error being within 1.5 %). This is due to the fact that, compared to the uniform leakage coefficient resulting from the classical model, the TIBERE model gives, in the voided zone, bigger leakage coefficients, and hence a bigger additional equivalent absorption. In the vicinity of the boundary of voided an non-voided zones, a greater discrepancy with respect to experiment appears; it could be explained by the fact that, in the collision probability calculation, azimuthal dependence in each cell was not taken into account; this insufficiency can have a particular effect in strong gradients zones. An interesting parameter is the ratio of maximum to minimum fission rate of the reactor. This value is 2.447 for the experiment, 2.369 for the classical leakage calculation (3.19 % relative error with respect to experiment) and 2.435 for the TIBERE calculation (0.49 % relative error with respect to experiment). As extension we have also calculated the same situation, but replacing aluminium by void. The decrease of reactivity between CRONOS with the classical procedure and CRONOS with TIBERE is now 284.10^{-5} (instead of 217.10^{-5} for aluminium). As far as the ratio of maximum to minimum fission rate is concerned, the classical procedure gives 2.257 and the TIBERE procedure 2.349; this shows that the relative discrepancy between these two calculation procedures is 3.92 % in the void case (instead of 2.71 % in the case of aluminium). However, the depression of fission rate is more important in the aluminium case, in sprite of the stronger streaming effect in void; this is due mainly to the fact that the absorption in Al is not entirely negligible. Then, in spite of its long m.f.p., Al cannot be considered exactly as void.

LEAKAGE COEFFICIENT CHARTS FOR VARIOUS TYPES OF VOIDED ASSEMBLIES

It seemed to us interesting to point out the difference between the leakage coefficient charts obtained by the procedures previously described. We present here the leakage coefficients of the perfectly reflected assembly (21 x 21 cells) used in the calculations of the EPICURE experiment. First we present the case where water is replaced by aluminium in the central zone (7 x 7 cells). Figs. 6 and 7 give, for the fast and the thermal group, the curves of the classical (D), axial (D_z) and radial (D_r) leakage coefficients as functions of the abcissa along the half diagonal of the assembly. These curves result from a two groups condensation of a 99 groups APOLLO-2 calculation of the currents j_{akk} and fluxes φ_s. Due to this spectrum consideration, the classical leakage coefficient $D = \frac{j_a}{\varphi_s}$ is not absolutely uniform (as it was before condensation), since the spectrum is space-dependent. On the contrary, the directional leakage coefficients given by the TIBERE procedure, $D_k = \frac{j_{akk}}{\varphi_s}$ are strongly space-dependent, due to the streaming in aluminium. The same calculation was performed by replacing aluminium by void (Figs. 8 and 9); the same effect appears, but is more important for directional coefficients (~ 30 to 50 %). For bigger voided zones (11 x 11, 15 x 15 cells), the effect is still greater. Moreover, for this type of reflected assembly (21 x 21 cells), the ratio of the maximum (at the cell corner) to minimum (at the center) fission rate of the assembly, for the voided zone of 7 x 7 cells, calculated by the classical APOLLO-2 procedure is 2.627 and by the TIBERE procedure 2.823. The same ratio for the voided zone of 11 x 11 cells, calculated by the classical APOLLO-2 procedure is 3.736 and by the TIBERE procedure 4 044. The relative discrepancy between the classical and the TIBERE procedure for the first case (7 x 7 voided cells) is 6.94 % and for the second one (11 x 11 voided cells) 7 62 %.

CONCLUSION

The new theoretical method, presented in this paper, implemented in APOLLO-2 code as the TIBERE procedure, allows to take into account, in 2D x-y fine group assembly calculations, the influence of assembly heterogeneity on neutron leakage. This method, based on the heterogeneous B_1 formalism, which assumes the existence of a fundamental mode, due to certain approximations, permits to obtain the space dependent leakage coefficients. Moreover, this new method requires tools which are very similar to those already existing in the APOLLO-2 code. In this way, it is, now, possible to define leakage cross sections, as additional absorption cross sections, which have space and energy dependence as well as all other cross sections. Hence, one obtains the new definition of the cell leakage rates. In order to perform, further on, the whole core calculation, one obtains, by means of the redefined transport-transport or transport-diffusion equivalence procedure, the cell homogenized cross sections where leakages are presented also as the cell homogenized leakage cross sections. The study of this refined heterogeneous leakage treatment was undertaken because of the insufficiency of the homogeneous leakage model, especially in cases when an assembly contains voided or almost voided zones, so that the streaming effect may become important. Meanwhile, as the other part of the improvement of knowledge of void effect, the series of experiments EPICURE was performed, where the water was substituted, in the central part of reactor, by aluminium in order to simulate void. The fission rate comparisons between the experimental results and the results of the whole reactor calculations were accomplished, where leakages were calculated by the classical (homogeneous) procedure and by the TIBERE (heterogeneous) procedure of APOLLO-2 code. They prove that the heterogeneous treatment of leakages give results which lie within the experimental error. This is not the case with the classical (homogeneous) leakage treatment. In other words, the classical leakage procedure underestimates the depression in the zone with aluminium due to streaming. It was shown also, that this effect was more exalted when the aluminium was replaced by void, and further on, much more exalted when the proportion of void was greater. This new model could be used, of course, for PWRs with other types of fuel (for example, MOX), and also for other types of reactors (for example: LMFBR's control assembly with or without sodium, HTGR).

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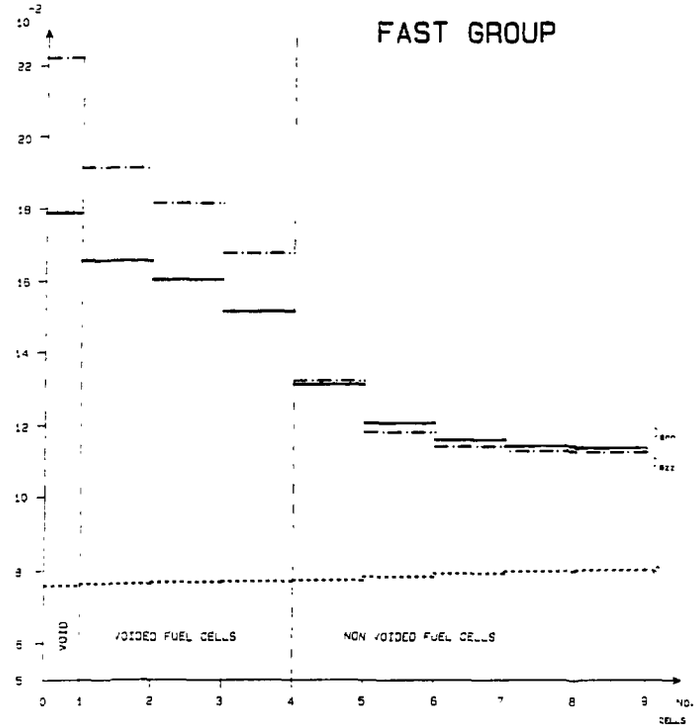

Figure 1: Fast group flux φ, axial j_{azz} and radial j_{arr} currents distributions in the half of the central 7 x 7 cells voided PWR assembly (17 x 17 cells).

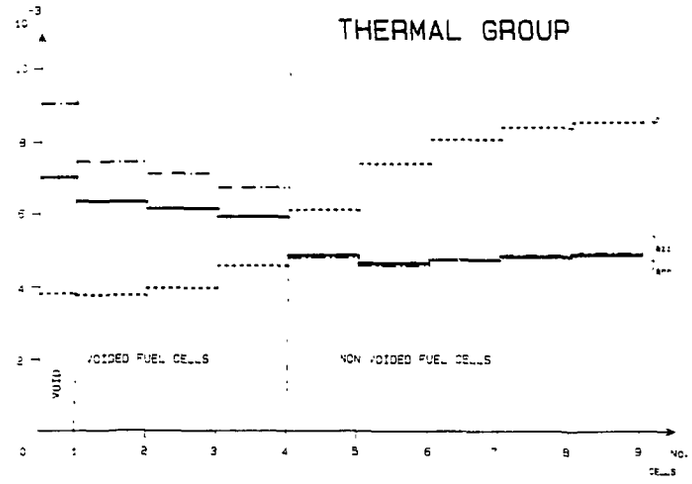

Figure 2: Thermal group flux φ, axial j_{azz} and radial j_{arr} currents distributions in the half of the central 7 x 7 cells voided PWR assembly (17 x 17 cells).

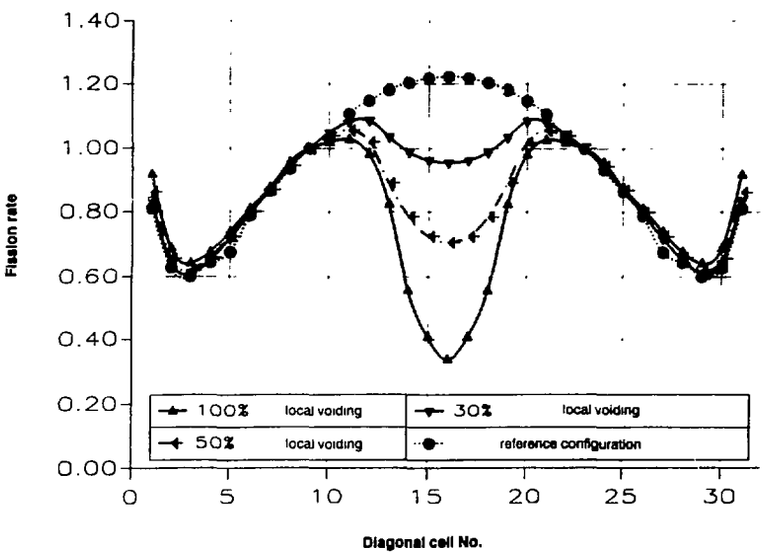

Figure 3: The approximate shape of the fission rate distributions in EPICURE voiding experiments.

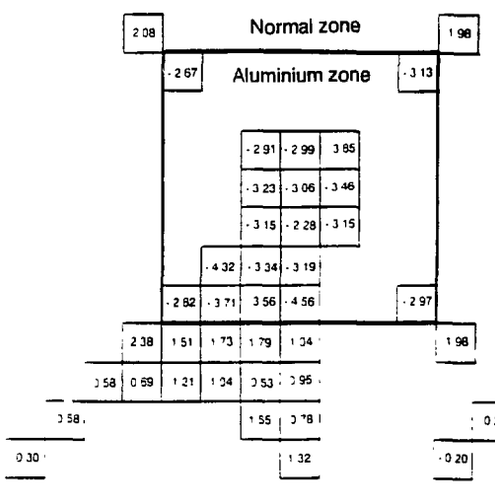

Figure 4: The relative discrepancy ((EXP - CAL/EXP)%) of the fission rate between the experiment and the classical leakage calculation.

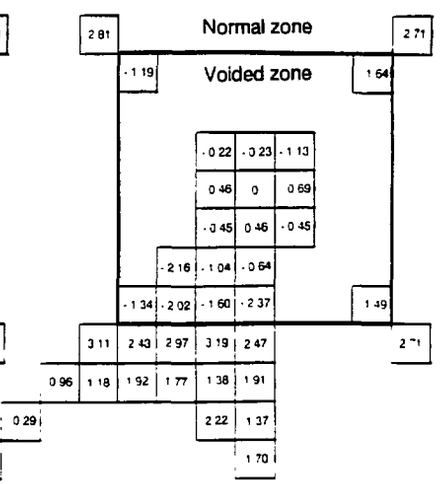

Figure 5: The relative discrepancy ((EXP - CAL/EXP)%) of the fission rate between the experiment and the TIBERE leakage calculation.

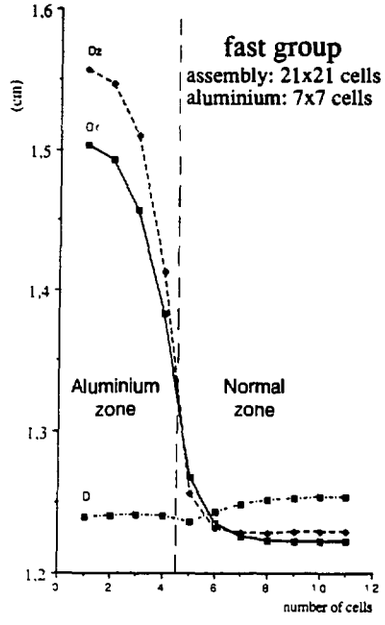

Figure 6: Leakage coefficients distributions.

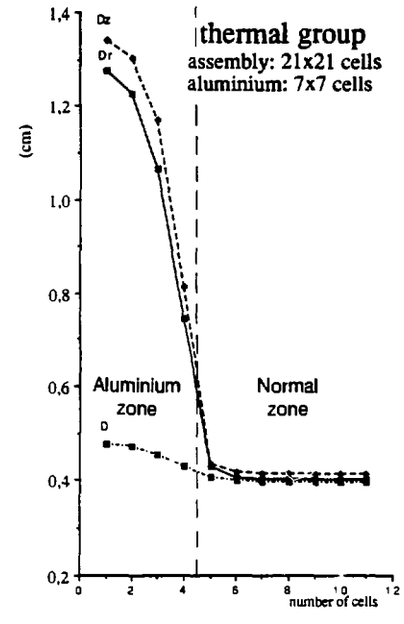

Figure 7: Leakage coefficients distributions.

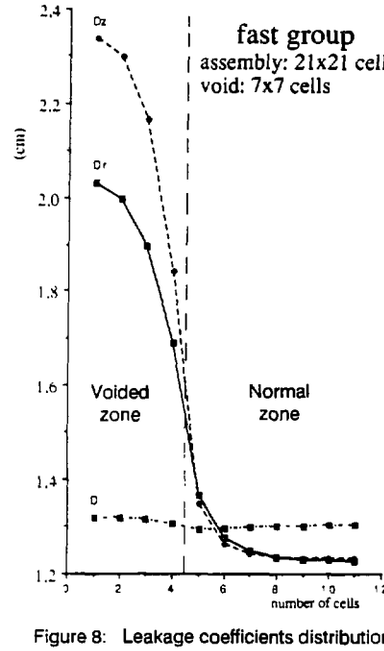

Figure 8: Leakage coefficients distributions.

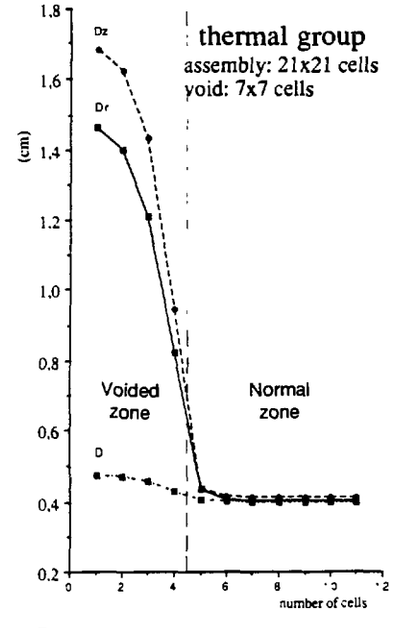

Figure 9: Leakage coefficients distributions.