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LARGE SIGNAL MODULATION OF SEMICONDUCTOR LASER AT HIGH BIT RATES

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Abstract – *In this paper we present numerical simulation of an InGaAs/InP heterostructure laser response to a digital modulation at high bit rates. The paper focuses on the influence of the nonlinear gain suppression and radiation recombination time on the relaxation oscillations. We find that relaxation oscillations might increase BER at high bit rates. The model which takes into account nonlinear gain suppression predicts better performances of the laser.*

1. INTRODUCTION

With sudden development, and sudden urge for utilizing optical networks for use in everyday life in the recent past, it has become necessary to make a deeper research into all its components, in order to achieve better performances. Optical transmitters play an important role in such systems, so optimizing their design is one of the key steps in building any optical system. Fiber-optic communication systems often use semiconductor optical sources, such as semiconductor lasers, because of several inherent advantages they offer. From the last 30 years of past century until now, semiconductor lasers have been developed extensively because of their importance for optical communications, medicine and many other potential applications.

In the purpose of better understanding the processes in semiconductor lasers, in this paper we investigate relaxation oscillations in an InGaAs/InP heterostructure laser, and its digital modulation. In comparison with the analog modulation, digital modulation has an advantage in the fact that it is less affected by various distortions of the optical signal during the transmission. This is an important fact since digital modulation is used for long-haul, large-capacity optical fiber communication systems. Indeed, in this paper we present and explain diagrams of a laser response to both the step impulse excitation and a series of rectangular impulses. In order to obtain these diagrams, we solved the rate equations represented by the system of two nonlinear first order differential equations. Since there is no exact analytical solution, it was necessary to solve them numerically. In addition, we give comparison of numerical results to approximate analytical solutions. In the next section we present a theory of the approach implemented in our calculation, based on the rate equations. Section 3 contains results of these calculations and discusses them. Final conclusions and a brief outlook are given in the Sec. 4.

2. THEORETICAL ANALYSIS

The rigorous derivation of the rate equations usually origins from *Maxwell's* equations together with quantum-mechanical approach for induced polarization. However, these equations can be written heuristically by considering various physical phenomena through which the number of photons, N_p, and the number of electrons, n, change with time inside the active region. According to this, and also assuming that electron concentration n is equal to hole concentration p, we can write these equations in the following form [1]:

$$\frac{dn}{dt} = \frac{\eta_i J}{qd} - \frac{n}{\tau} - \Omega \cdot \left(n - n_{nom}\right) \cdot \frac{N_p}{1 + \varepsilon \cdot N_p} \tag{1}$$

$$\frac{dN_p}{dt} = \Gamma \cdot \Omega \cdot \left(n - n_{nom}\right) \cdot \frac{N_p}{1 + \varepsilon \cdot N_p} + \frac{\Gamma \cdot \theta \cdot n}{\tau_r} - \frac{N_p}{\tau_p} \tag{2}$$

where J is injection current density, q stands for electron charge and d is thickness of active area. $\eta_i=\tau_{nr}/(\tau_r+\tau_{nr})$ is internal quantum efficiency [1] which depends on radiative and nonradiative recombination time τ_r and τ_{nr}, respectively. Furthermore, τ represents electron lifetime, differential gain is given by $\Omega=c/n_r \cdot \partial g/\partial n$, with g representing the material gain. Transparency electron concentration is represented by n_{nom}, Γ is optical confinement factor of the active layer, θ is spontaneous emission coupling factor which is defined as the ratio of spontaneous emission coupling rate due to lasing mode to total spontaneous emission rate [1] and τ_p stands for photon life time. In the purpose of obtaining more accurate results and realistic model of physical processes in heterostructure lasers, we include nonlinear gain suppression (NGS) coefficient denoted with ε [2], in Eq. (1) and (2).

Looking at the rate equations (1) and (2), it is clear that one should calculate Γ, g and τ_p in order to solve the system. In order to calculate confinement factor we solved *Helmholtz* equations for planar dielectric waveguide [3] using finite difference method.

Material gain is calculated using [2]:

$$g(E) = -const \cdot E \cdot \sqrt{E - E_g} \cdot \left(\frac{2m_r}{\hbar^2}\right)^{3/2} \cdot \left[f_v\left(-E_g - \frac{m_c(E-E_g)}{m_c+m_v}\right) - f_c\left(\frac{m_v(E-E_g)}{m_c+m_v}\right)\right] \tag{3}$$

with $const=q^2 \cdot h \cdot |M_b|^2/4m_0 \cdot c \cdot \varepsilon_0 \cdot \pi^2$, where h is Planck's constant, f_v and f_c are Fermi-Dirac distribution functions for quasi Fermi states in valence and conductive band respectively. $|M_b|$ is Kane's matrix element [1], E is energy of incident photon, E_g is active region energy gap, and m_r is reduced mass.

Photon lifetime is the third very important term of the rate equations, calculated as $\tau_p^{-1}=\Gamma v_g g_{th} - \varepsilon\Gamma(J-J_{th})2d/(qLw)$ [2], where v_g stands for group velocity defined as c/n_r and g_{th} and J_{th} stand for threshold gain and current, respectively. Dimensions of active layer are defined with L and w, as length and width of active layer, respectively.

When a semiconductor laser is modulated, because it takes time for a carrier population to build up, there will be a certain time delay before the final photon density is reached.

Once this steady state is achieved, additional time is required for both carrier (electron-hole pairs) and photon population to come into equilibrium. This phenomenon, called relaxation oscillations, can be predicted even without numerically solving the rate equations. If we assume the small signal analysis, we can obtain approximate analytical solutions that give us clear physical meaning of the process. First, if we consider laser in the region of spontaneous emission, while $J<J_{th}$ or $n<n_{th}=n^{av}$, rate equation (1) collapses to, very simple first order differential equation, easy to solve. From the solution of this equation, when $n(t)$ reaches n_{th}, we can calculate turn-on delay time t_d, as:

$$t_d = \tau \ln\left(\frac{J}{J-J_{th}}\right). \tag{4}$$

For $t>t_d$ we have stimulated emission, and in order to approximately solve the rate equations we assume $n=n^{av}+\Delta n$ considering that $n^{av}>>\Delta n$, and similarly for photons: $N_p=N_p^{av}+\Delta N_p$ also considering $N_p^{av}>>\Delta N_p$. Implementing this into the final form of the rate equations (1) and (2), neglecting nonlinear gain suppression for the sake of simplicity, and considering that for steady state $dn^{av}/dt=0$ and $dN_p^{av}/dt=0$, we obtain two independent differential equations, that can be solved analytically. Solution takes form:

$$\Delta n(t) = \Delta n_0 \cdot e^{-Q\cdot t} \sin(\omega_r t + \varphi_n), \tag{5}$$

$$\Delta N_p(t) = \Delta N_p \cdot e^{-Q\cdot t} \sin(\omega_r t + \varphi_n), \tag{6}$$

with:

$$Q = \frac{1}{2}\left(\Omega N_p^{av} + \frac{1}{\tau}\right), \tag{7}$$

$$\omega_r = \sqrt{\left(\frac{\Omega N_p^{av}}{\tau_p} + \frac{\theta}{\tau_r \tau_p}\right) - Q^2} \tag{8}$$

standing for oscillations frequency and decay coefficient, respectively. Finally, average photon density is calculated as $N_p^{av} = \Gamma(J-J_{th})\tau_p/(qd)$ [1].

3. RESULTS AND DISCUSSION

Results in this section are presented for the heterostructure $In_{0.53}Ga_{0.47}As/InP$ laser, with active layer thickness of d=0.2μm with length and width: L=300μm and w=2μm, respectively. It is assumed that losses in these two layers are $\alpha_a=30cm^{-1}$ and $\alpha_c=5cm^{-1}$, respectively. Reflectivities of resonator both mirrors are evaluated to be 0.32, according to *Fresnel* equations [3]. Electron life time is considered to be 2.71ns [2]. Refractive index is calculated to be n_{ra}=3.61 and n_{rc}=3.32 for active layer and cladding, respectively. Theoreticaly ideal model is considered, with no parasitics, and relieved from the effects of the enviroment. Spontaneous emission factor [1] showed to be extremely important term in numerical treating of the problem. Although it can be neglected in analytical solutions without leading to miscalculation, it is necessary to calculate it in when numerically solving the system (1)-(2), since it forms boundary condition and „triggers“ the equations. For our structure we estimated the value of this factor using [1]:

$$\theta = q\frac{c}{n_r}\frac{\Gamma g_{th} n_{sp}}{\eta_i \eta_r J L w}, \tag{9}$$

where J, as used here, does not include stimulated emission current, so we took J_{th} as the value of J. Population inversion factor n_{sp} is usually between 1.25 and 1.75 [1] for gain range commonly required in lasers, and nonradiative recombination is assumed to be minimal, hence η_r=1 can be assumed. Threshold gain is calculated from Eq. (3), and it takes value of $1.1485\cdot10^4 m^{-1}$. Typically, spontaneous emission factor should be in order of 10^{-5} [1], indeed we calculated $0.55\cdot10^{-5}$, what is compatible with theoretical expectations.

Since the rate equations (1) and (2) cannot be solved analytically, we solved the equation system numerically. As the result, we present diagrams of electron-hole pairs (carrier) density n versus time, and photon density N_p versus time, on Fig. 1.

Fig.1, shows laser relaxation oscillations, for different values of radiative recombination time. Injected current density is unique for all three cases and takes value of $J=10^8 A/m^2$, and turn-on time for this current is t=0. Dashed line on Fig.1 represents carrier density versus time, respectively, for material with radiative recombination time of 3ns, without nonlinear gain suppression included. When it is compared to adequate solid line, representing carrier density with NGS, described by coefficient $\varepsilon=2.4\cdot10^{-17}cm^{-3}$,

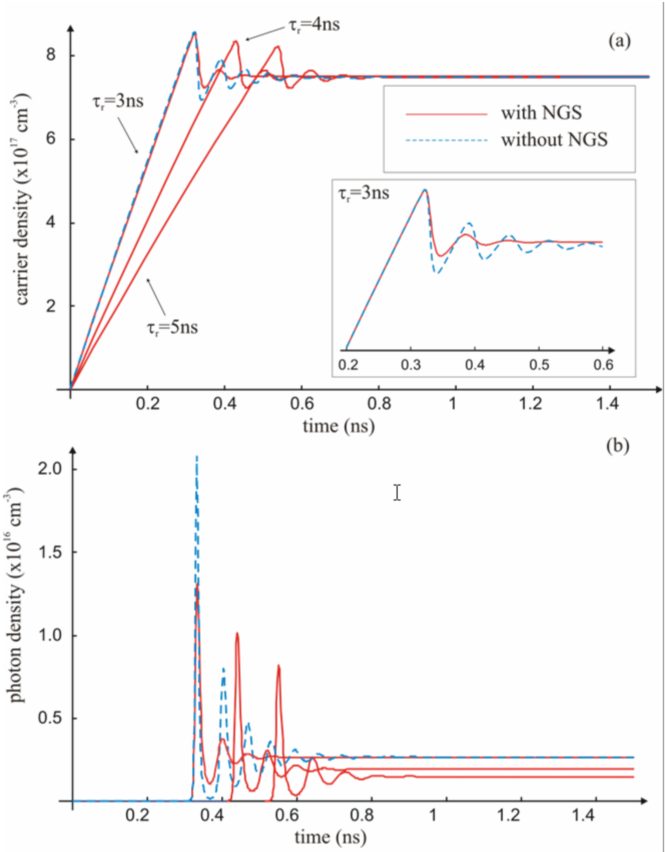

Fig.1. *(a) Carrier and (b) photon response on Heaviside excitation for different values of radiative recombination time, with (solid lines) and without (dashed lines) nonlinear gain suppression (NGS) included. The insert in (a) represents enlarged time interval between 0.2-0.6 ns.*

it is observed that taking into account NGS, does not affect turn-on delay time, and slightly affects relaxation oscillation. However, calculating NGS in, significantly changes decay in amplitude of oscillation peaks, therefore effect of NGS cannot be neglected in further calculations.

The influence of changing radiative recombination time is presented on Fig. 1. Increasing radiative recombination time increases turn-on delay time and decreasing photon density steady-state value at the same time. This coincides with theory, because, by changing radiative recombination time, we change the internal quantum efficiency in the manner described in the Sec. 2. Therefore, increasing radiative recombination time results with quantum efficiency decrease, introducing the same effect as reducing the injection current density. For example turn-on delay time t_d increases while steady-state value of photon density decreases when we reduce the injection current density. Therefore, the same effect appears with an increase of radiative recombination time. Numerical data considering this are given in the Table 1. Also, radiative recombination time has an influence on spontaneous emission coupling factor, and while τ_r increases, θ increases as well.

Numerical calculations of decay coefficient, oscillations frequency and turn-on delay time are given in Table 1. In addition, this table contains comparison of this to the analytically calculated results. Using the small signal analysis, according to (4), (7) and (8) we obtained analytical solutions for these parameters. In this calculation NGS was neglected for the simplicity of the procedure.

According to the data in Table 1, it can be observed that all three parameters (decay, frequency and delay time) show great dependance of radiative recombination time, no matter with or without NGS included. Decay coefficient Q and relaxation oscillations frequency ω_r decrease practically linearly with increase of radiative recombination time, while turn-on delay time t_d changes in the manner explained before. Also, taking into account NGS, practically, does not change at all turn-on delay time, and does not signifficantly affects on ω_r. However, it increases Q for more than 2 times, because of which carrier and photon density reach steady-state much quicker. This is very important result, especially when it comes to calculating large signal modulation at high bit rates. Also, comparing numerically to analytically extracted data, we observe that difference between them is too large, mainly because analytics show no dependance of any parameter of radiative recombination time and, also because effect of NGS is neglected in analytical derivations and calculations.

Table 1. *Numeric to theory comparison of decay, frequency and delay time, with and without NGS included.*

τ_r (ns)	$Q(\cdot 10^{10}$ 1/s)	ω_r ($\cdot 10^{10}$ rad/s)	t_d ($\cdot 10^{-10}$ s)
numerical (without NGS)			
3	1.1375	8.3888	2.8390
4	0.7950	6.8895	3.8550
5	0.5989	5.8997	4.9110
numerical (with NGS)			
3	2.9044	9.2190	2.8720
4	2.1146	7.8442	3.8880
5	1.6251	6.8444	4.9440
analytical			
3, 4, 5	1.0390	9.5971	4.2904

Second important discussion considers the response of the heterostructure laser, on series of rectangular impulses for couple of cases of bit rate B (2.5 Gb/s and 10 Gb/s) with both NGS included and not. Also, we considered two cases of modulation, one with logical „0“ set at $J= 0.95J_{th}$ and second with $J=1.2J_{th.}$ These results are shown on Figs. 2 through 5.

In order to achieve high-speed optical signals, the optical pulses have to quickly return to their steady state values. Therefore, the decay coefficient and oscillation frequency have to be large. According to the Table 1, we can infer that with decreasing radiative recombination time we can obtain larger Q and ω_r. Therefore, Figs. 2 through 5 are obtained with τ_r considered to be 3 ns. Although turn-on delay time is smaller when logical "0" is set at 1.2 J_{th} in this case we have oscillations around logical "0" level, what increases BER (bit error rate). However, for extremely large bit rate flows, it is possible to hold logical "0" a little above J_{th} avoiding any oscillations as it is seen in Fig. 5. It can also be implied, that when the modulation speed is high, the pulse width is very small, and in that case the number of peaks in relaxation oscillations is reduced, and a steady state does not exist within each pulse (c.f. 4 and 5). Hence, a decrease in average light intensity and multimode operation takes place. The solution for this is again increasing Q and ω_r, in order to achieve steady state within every pulse.

4. CONCLUSION

In this paper we analyze the influence of nonlinear gain suppression and radiative recombination time on the

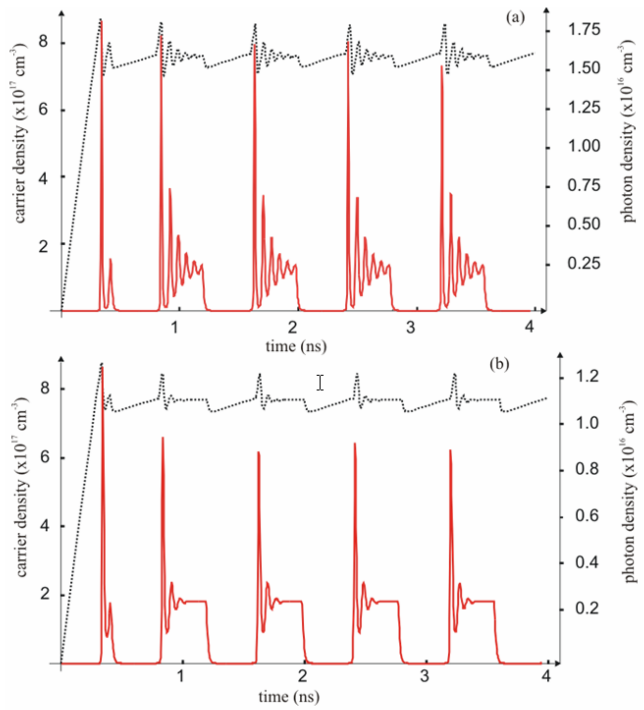

Fig.2. *Laser response for B = 2.5 Gb/s with logical „0“ set at J =0.95 J_{th} and „1“ set at J = 10^8 A/m^2, (a) without and (b) with NGS included.*

oscillations in the heterostructure laser. It can be seen that, when NGS included, we obtain stronger decay coefficient and oscillations frequency, resulting in better laser dynamics performances. This is particularly significant during high bit rate modulation, since it allows laser to reach the steady state faster.

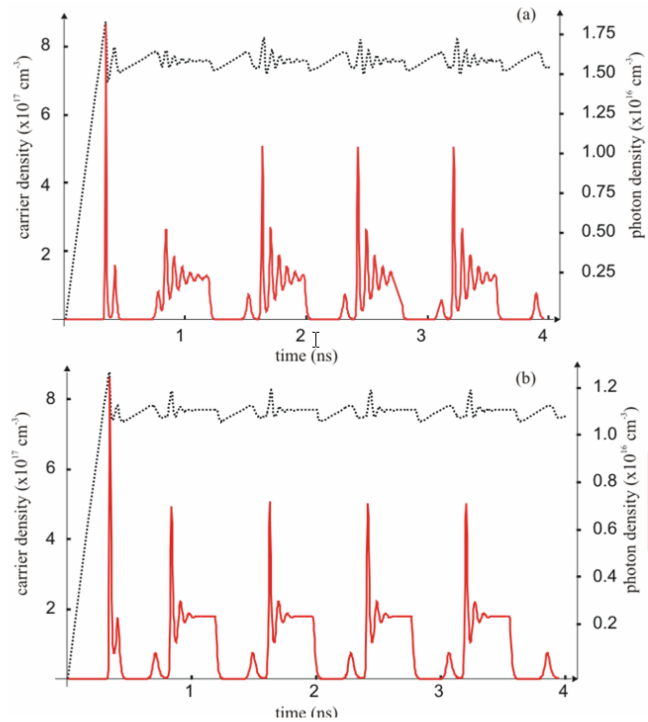

Fig.3. Laser response for B = 2.5 Gb/s with logical „0“ set at J =1.2 J_{th} and „1“ set at J = 10^8 A/m^2, (a) without and (b) with NGS included.

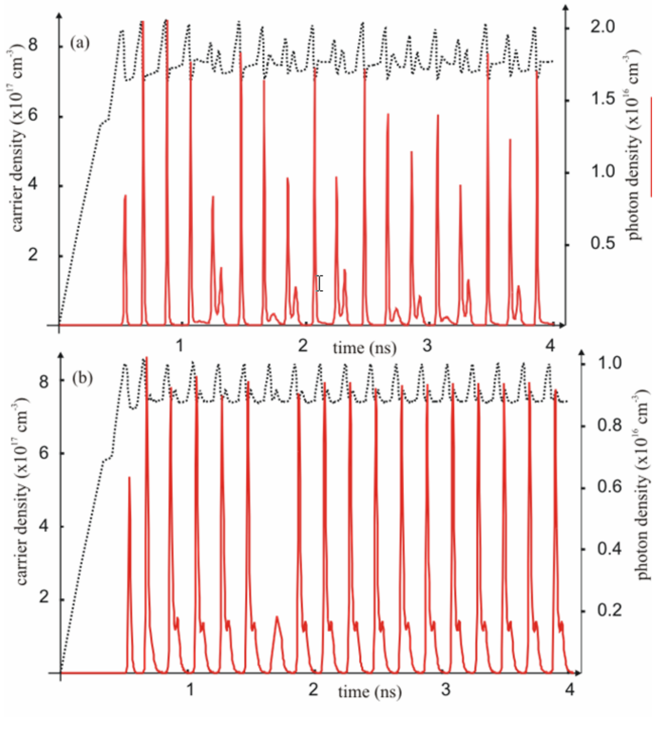

Fig.4. *Laser response for B = 10 Gb/s with logical „0“ set at J =0.95 J_{th} and „1“ set at J = 10^8 A/m^2,(a) without and (b) with NGS included).*

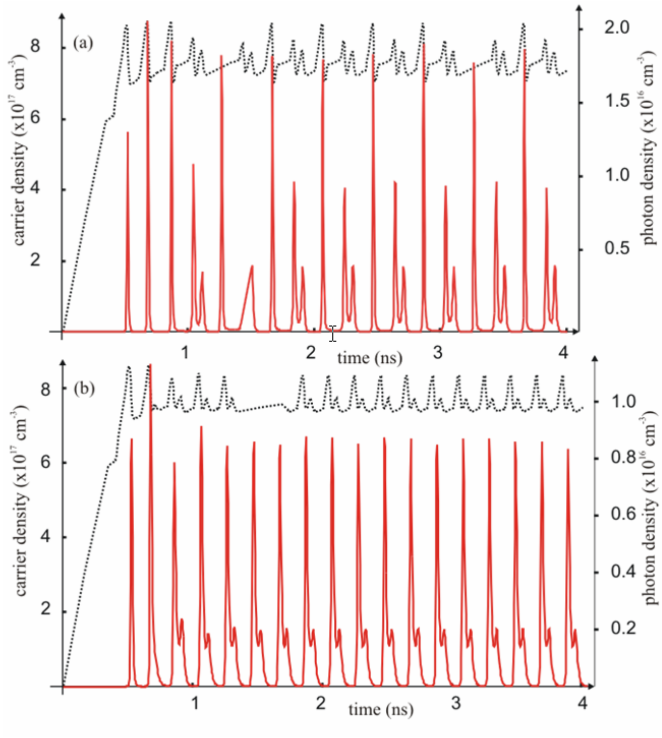

Fig.5. *Laser response for B = 2.5 Gb/s with logical „0“ set at J =1.2J_{th} and „1“ set at J = 10^8 A/m^2, (a) without and (b) with NGS included.*

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Sadržaj – U ovom radu predstavljena je numerička simulacija fenomena prelaznog režima uključenja poluprovodničkog heterostrukturnog lasera na bazi InGaAsP/InP, pri visokim bitskim protocima. Rad istražuje uticaj nelinearnog faktora potiskivanja i vremena rekombinacije na relaksacione oscilacije kod heterostrukturnih lasera. U razmatranju ovog problema pokazalo se da relaksacione oscilacije mogu da povećaju BER pri velikim bitskim protocima. Model koji u obzir uzima nelinearni faktor potiskivanja predviđa bolje performanse lasera.

MODULACIJA POLUPROVODNIČKOG LASERA SIGNALIMA VELIKIH BITSKIH BRZINA

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