ISSN 1450-7404

Journal of Research in Physics

Volume 31
Number 2
2007

DEPARTMENT OF PHYSICS

Faculty of Sciences
University of Novi Sad - Novi Sad - Serbia

Coincidence summing effects in gamma-ray spectrometry

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Abstract
A new method for theoretical calculation of coincidence summing of X- and gamma rays has been described. This model enables forming the count rate equations for single, summing peaks and total count rate. The detector efficiencies and the activity of the source were obtained by solving of the system count rate equations. The method was experimentally confirmed at the decays of ^{57}Co and ^{133}Ba.

1. Introduction

Coincidence summing of X- and gamma-rays in spectra recorded by HPGe detectors is one of the most important problems in gamma ray spectrometry. Many authors paid attention to this problem, but the most frequent approach is so called matrix formalism, developed by Semkow et al. [1] for gamma-gamma coincidence summing and Korun and Martinčić method [2] for X-gamma coincidence summing. However, all coincidence summing are not involved by Semkow et al. method if the daughter nucleus has more than two excited levels. Korun and Martinčić method [2] has the same disadvantages since their method is an extension of the matrix formalism to X-rays. It can be applied only to radionuclides the daughter nucleus of which has only one excited level. They successfully applied their method to ^{139}Ce decay, but its daughter nucleus has only one excited level.

2. Transition probability matrix

The theoretical model for coincidence summing developed in ref. [3] and applied in ref. [4] successfully solved these problems. Using this method, all decay paths can easily be identified, and all decay paths outcomes can be determined using simple mathematical program. One decay path outcome is characterized by two quantities, by the deposited energy in the detector and by the probability for that. The sum of all probabilities of particular outcomes having the same energy is the total probability per one decay of appearing a peak of that energy in the recorded spectrum. The product of the activity of the measured source and total probability gives the peak count rate equation.

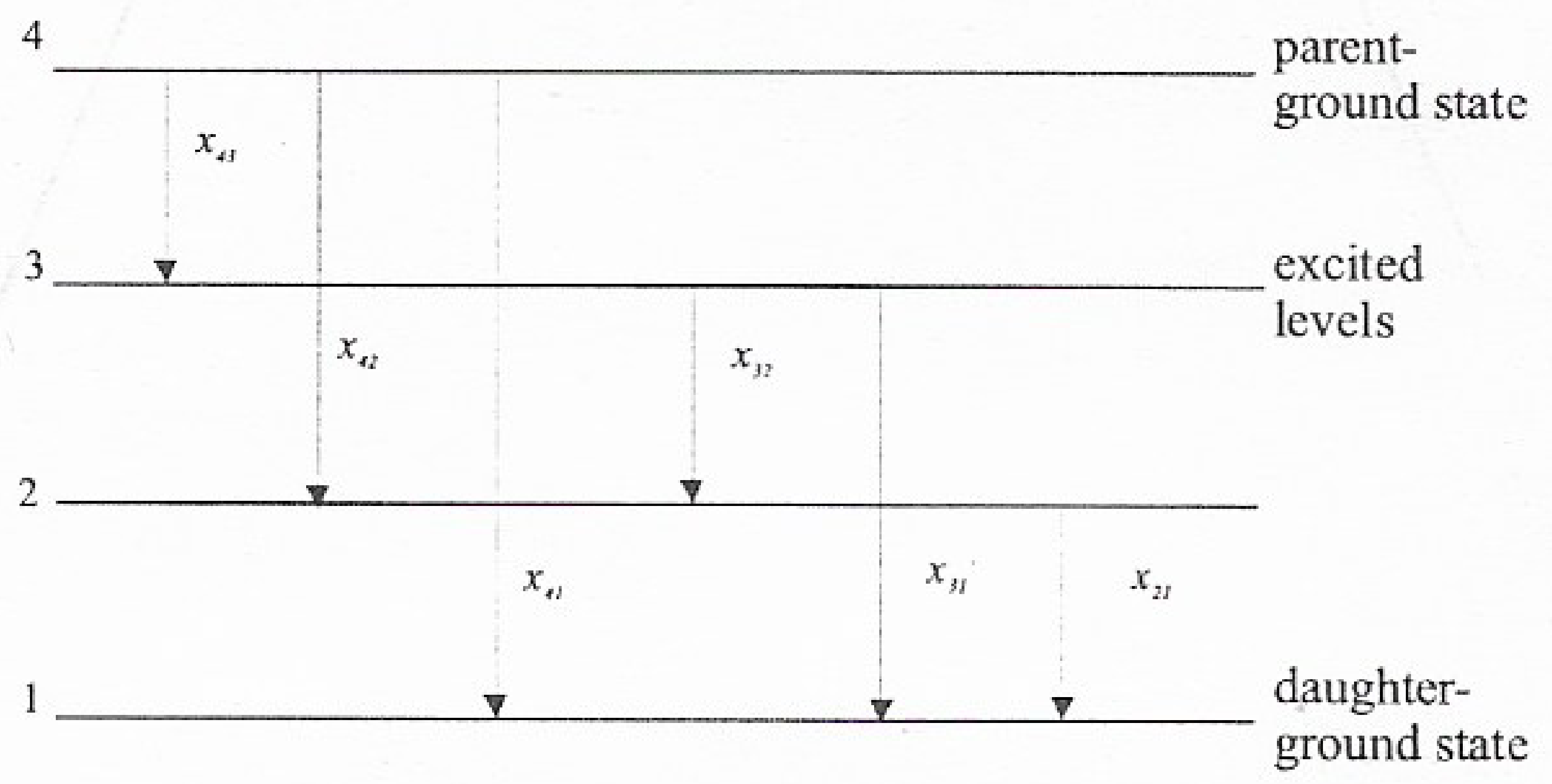

Fig. 1. General decay scheme.

To illustrate this theory we shall start from the decay scheme representing a nucleus with two excited levels. The parent ground state decays with electron capture into a daughter nucleus, which is presented as the third daughter excited level (Figure 1). The transition probability matrix corresponding to this modified scheme, **X**, is

$$\mathbf{X} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ x_{21} & 0 & 0 & 0 \\ x_{31} & x_{32} & 0 & 0 \\ x_{41} & x_{42} & x_{43} & 0 \end{bmatrix} \qquad (1)$$

where x_{4i} (i=1,2 and 3) represents probabilities of the transitions of parent ground state by electron capture to the daughter excited levels, and x_{ij} (i=2,3; j=1,2) represents the normalized probability for transition from level i to level j for the daughter nucleus. Matrix element $[X]_{41}=x_{41}$ represents the probability for the decay in one step. Matrix element $[X^2]_{41}=x_{42}x_{21}+x_{43}x_{31}$ represents the probability for the decay in two steps. The elements $x_{42}x_{21}$ and $x_{43}x_{31}$ represent probabilities for the decay with cascade transitions 4→2→1 and 4→3→1, respectively. Finally, $[X^3]_{41}=x_{43}x_{32}x_{21}$ represents a probability for the decay in the three steps with the cascade transitions 4→3→2→1. The matrix X^n (n>3) has all matrix elements equal zero, which means that there is not decay in four steps. The sum of all matrix elements is equal 1, because transition from the ground state of the parent nucleus to the ground state of the daughter nucleus is a certain event.

3. Decay paths

The decay path is defined by the cascade transitions from the parent ground state to the daughter ground state. In order to determine the decay paths, we introduce the matrix **Y**,

$$\mathbf{Y} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ y_{21} & 0 & 0 & 0 \\ y_{31} & y_{32} & 0 & 0 \\ y_{41} & y_{42} & y_{43} & 0 \end{bmatrix}, \qquad (2)$$

where matrix element yij represents the transition from the i^{th} level to the j^{th} level. Matrix element $[\mathbf{Y}]_{41}=y_{41}$ is the symbol of the transition from level 4 to level 1 in one step. The matrix element $[\mathbf{Y}^2]_{41}=y_{42}y_{21}+y_{43}y_{31}$ consists of two terms and it means that the transitions 4→1 in two steps can happen in two ways, 4→3→1 and 4→2→1. The third power of matrix **Y** contains only one non-zero element, $y_{43}y_{32}y_{21}$. The transition 4→1 in three steps occurs in the cascade 4→3→2→1.

4. Decay path outcomes

From the point of view of the spectrometric detection of X- and gamma rays, emitted in the radionuclide decay, each level is the point of branching because the consequences of one transition can be different. The transition from levels 4 to lower level (electron capture) is followed by one of the three possible events: by detection of K_α or K_β photon emitted at the electron capture, or without detection of any photon. The transition from the daughter level 3 and 2 to the lower daughter level is followed by one of the four events: by detection of K_α, K_β photon from an internal conversion, by detection of the gamma photon, or without detection of any photon. Therefore, matrix elements y_{4i} (i<4) and y_{ij} (j<i<4) are replaced by matrices-rows of three and four elements, respectively,

$$\begin{aligned} y_{4i} &= [p_{\alpha 4i}, p_{\beta 4i}, q_{4i}] \quad i<4, \\ y_{ij} &= [p_{\alpha ij}, p_{\beta ij}, \gamma_{ij}, q_{ij}] \quad ,j<i<4 \end{aligned} \qquad (3)$$

where $p_{\alpha 4i}$ and $p_{\beta 4i}$ are the probabilities of detection of K_α and K_β photon in the electron capture process (transition 4→i), respectively, and q_{4i} is the non-detection probability of any photon. The probabilities of detection of K_α and K_β photons in the internal conversion process (transition i→j) are denote in the equation (3) as $p_{\alpha ij}$ and $p_{\beta ij}$, respectively, while γ_{ij} denotes probability of detection of the gamma photon emitted at the transition i→j and q_{ij} is the non-detection probability of any photon. The number of decay path outcomes is enlarged three times at 4→i transition and four times at i→j transition. In order to comprehend all of these possibilities, the matrices in decay path are directly (Kronecker`s) multiplied. For example, decay path in the two steps gave a new matrix-row,

$$y_{4i}y_{ij} \Rightarrow y_{4i} \otimes y_{ij} =$$

$$[p_{\alpha 4i}\,p_{\alpha ij}, p_{\alpha 4i}\,p_{\beta ij}, p_{\alpha 4i}\,\gamma_{ij}, p_{\alpha 4i}\,q_{ij}, p_{\beta 4i}\,p_{\alpha ij}, p_{\beta 4i}\,p_{\beta ij}, p_{\beta 4i}\,\gamma_{ij}, p_{\beta 4i}\,q_{ij}, q_{4i}\,p_{\alpha ij}, q_{4i}\,p_{\beta ij}, q_{4i}\,\gamma_{ij}, q_{4i}\,q_{ij}], \tag{4}$$

which has 3×4=12 elements. The probabilities of all outcomes in the two steps cascade transition 4→i→j are contained in list (4). The lists of probabilities (3) should be correlated with the possible depositions of the energies of the emitted photons in the detector. Therefore, we introduce a new matrix-row,

$$z_{4i} = \{E_\alpha, E_\beta, 0\}, \quad i < 4,$$

$$z_{ij} = \{E_\alpha, E_\beta, E_{\gamma ij}, 0\}, \quad j < i < 4, \tag{5}$$

where E_α and E_β are energies of K_α and K_β photons, respectively, emitted at electron capture or internal conversion, $E_{\gamma ij}$ is the photon energy emitted at the transition $i \rightarrow j$, while 0 denotes that there is no detection. Similarly as in equation (4) we can apply direct (Kronecker`s) summing on z_{4i} and z_{ij},

$$z_{4i}z_{ij} \Rightarrow z_{4i} \oplus z_{ij} =$$

$$[2\,E_\alpha, E_\alpha+E_\beta\,, E_\alpha+E_{\gamma ij}, E_\alpha\,, E_\beta+ E_\alpha, 2\,E_\beta, E_\beta+E_{\gamma ij}, E_\beta\,, E_\alpha\,, E_\beta\,, E_{\gamma ij}\,, 0]. \tag{6}$$

A new matrix-row with 12 elements, i.e. 12 deposited energies in the detector is obtained. The position of the deposited energy in eq. (6) and the position of the probability in eq. (3), corelating with deposited energy, are same. In the given excample, the decay path has 12 decay path outcomes, and each of them is characterized by two quanteties, by a deposited energy in the detector and by a probability for that event.

5. The grouping of the elements of the matrix E by energy outcome

Matrix elements $[\mathbf{Y}^k]_{n1}$ (k=1, 2, ..., n-1) are tools for finding all decay paths. One decay path, representing radionuclide disintegration in k steps, contains k matrix elements of matrix **Y**, which are to be substituted by matrices (3) and direct multiplied as shown by eq. (4). The result of this multiplication is a new matrix-row of $3\times 4^{k-1}$ elements. The number of decay path outcomes amounts $3\times 4^{k-1}$ only for one decay path of the k steps. If one disintegration contains m decay paths, this procedure is repeated m times and the result is m new matrices which are to be joined in a united matrix **P**,

$$\mathbf{P} = [p_1, p_2, \ldots\ , p_N], \tag{7}$$

the elements of which are the probabilities of particular outcomes. However, one decay path outcome is characterized by two quantities, by the deposited energy in the detector and by a probability for that. In order to obtain a matrix-row of the energies deposited in the detector for all decay path outcomes, we perform direct summing of energy matrices z_{ni} and z_{ij}. The resulting matrix- row **E**, has the form,

$$\mathbf{E} = [e_1, e_2, \ldots\ , e_N]. \tag{8}$$

Since the multiplication of the probability matrices and summing of the energy matrices were performed in the same order, the correlation between the deposited energies and probabilities for

deposition of those energies are saved. The energy matrix **E** has the same values at different positions in the matrix, which means that the same energy deposition can be the result of different decay path outcomes. The sum of all elements in **P** having the same energy in matrix **E,** is the total probability per one decay of appearing a peak of that energy in the recorded spectrum. The product of the activity of the measured source and total probability gives the peak count rate equation. The grouping of the elements of **P** with the same energy deposition is performed by the positions of that energy in matrix **E**.
The probabilities given in matrices (3) can be defined with the known nuclear and atomic parameters, and with the unknown peak and total efficiencies. The nuclear and atomic data which determine these probabilities are the internal conversion coefficients, the K-shell fluorescent yield and the probability of the K-shell electron capture. These data are available in the literature [5]. Thus defined probabilities enable forming the equation for the count rate of every peak and also for the total count rate. The unknown variables in these equations are peak and total efficiencies, which can be determined from the recorded spectrum.

6. Summary and conclusion

The previous method described in the short features, was successfully applied to ^{139}Ce, ^{57}Co and ^{133}Ba decay. This method enables us to find all decay paths, all decay path outcomes, and to form count rate equations for all summing peaks, single peaks, as well as for the total count rate. The radionuclide with a relatively simple decay scheme like ^{57}Co has 60 decay path outcomes and 25 count rate equations. ^{133}Ba has 11 decay paths, 1788 decay path outcomes and 168 count rate equations. The equaling theoretical expressions for count rate and experimental values for the most important peaks, the count rate equations are obtained. The coefficients in the equations are the nuclear and atomic parameters of the radionuclide. Efficiency curve (Figure 2) and activity of the point source are determined by solving the system of count rate equations.

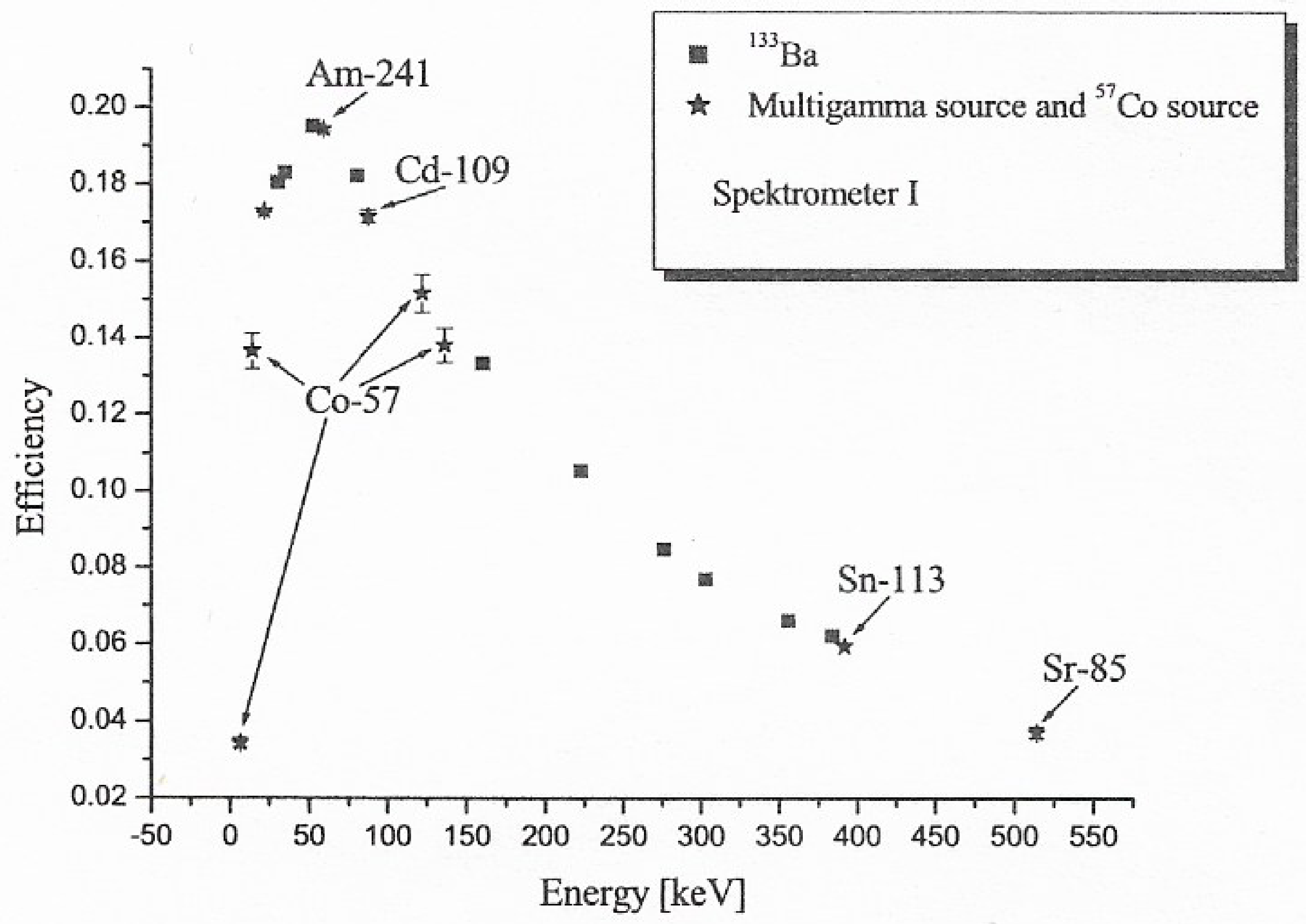

Figure 2. Efficiency versus energy.

The most important characteristic of this method is possibility to foresee qualitatively and quantitatively all coincidence summing peaks and total count. There are not any limitations about the complexity of the decay scheme for the study of the radionuclide. Next step in the development of this method is applying it to the volume sources.

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