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Temperature effects on fidelity of reflection from absorbing Bragg mirrors

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Abstract. In the paper, we consider the quantum effects in reflection of the electromagnetic field from an absorbing Bragg mirror. The special emphasis is placed on the study of the influence of thermal noises of the mirror environment on the fidelity of reflection. Taking the incident light field in coherent and Fock states, we show that the fidelity of reflection turns out to be strongly dependent on the quantum state, and on the level of thermal noise and absorption of the mirror.

1. Introduction

A Bragg mirror (or distributed Bragg reflector) is an optical device which is composed of an alternating sequence of multiple layers of two different materials. Each optical layer thickness corresponds to one quarter of the wavelength that provides constructive interference of the reflected waves from the mirror layers. There thus appears a certain range of wavelengths (called the photonic stop band), within which light propagation in the mirror structure is strongly suppressed. It is why Bragg mirrors act as high-quality reflectors, so that they have a wide use in many optical devices, such as light-emitting diodes [1], vertical cavity surface emitting lasers [2], solar cells [3], and Fabry-Pérot filters [4].

However, in the case of the quantized electromagnetic fields the mirror reflectivity, which is just related to intensities of the incident and reflected fields, cannot fully characterize the process of reflection. This is due to the fact that the interaction of the incident electromagnetic field with the mirror materials might cause a significant change in the quantum state of reflected light. Instead, the fidelity of the incident and reflected states of light that shows their quantum closeness may serve as the measure of the reflection efficiency.

The present report is aimed to investigate the effects of reflection of quantum states from a Bragg mirror in view of thermal noises of its environment. Such an analysis looks to be important for the quantum information processing of light states with the use of optical cavities [5].

2. The Model

From the theoretical point of view a Bragg reflector can be considered as a four-port device formed as a sequence of different semiconductor and dielectric plates of the certain refractive indices (figure 1). Our description of the quantum-state transformation of the fields by the Bragg reflector is based on the input-output relations at dispersive and absorbing four-port

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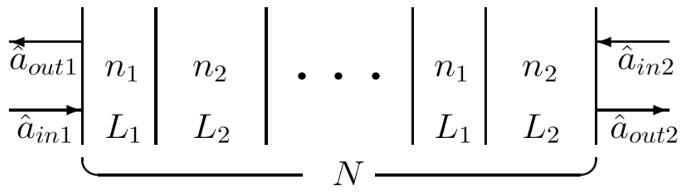


Figure 1. Four-port scheme for a N-stack Bragg mirror.

devices [6, 7]. These relations were derived by using the Green-function approach for quantization of the electromagnetic field in dispersive and absorbing media and have the form [6]

$$\begin{pmatrix} \hat{a}_{out1}(\omega) \\ \hat{a}_{out2}(\omega) \end{pmatrix} = \mathbf{T}(\omega) \begin{pmatrix} \hat{a}_{in1}(\omega) \\ \hat{a}_{in2}(\omega) \end{pmatrix} + \mathbf{A}(\omega) \begin{pmatrix} \hat{g}_{+}(\omega) \\ \hat{g}_{-}(\omega) \end{pmatrix}. \tag{1}$$

In equation (1) $\hat{a}_{in1,2}$ and $\hat{a}_{out1,2}$ stand for annihilation operators of the incoming and outgoing fields, respectively. The bosonic annihilation operators $\hat{g}_{\pm}$ describe excitations associated with the device and play the role of the additional operator noise sources. The 2×2 matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ are the characteristic transformation and absorption matrices of the device. When the device is surrounded by vacuum, then the the matrices $\mathbf{T}(\omega)$ and $\mathbf{A}(\omega)$ obey the relation

$$\mathbf{T}(\omega)\mathbf{T}^{\dagger}(\omega) + \mathbf{A}(\omega)\mathbf{A}^{\dagger}(\omega) = \mathbf{I}, \tag{2}$$

where $\mathbf{I}$ is a unity 2×2 matrix. In such notations, $\mathcal{R} = |T_{11}(\omega)|^2$ and $\mathcal{T} = |T_{12}(\omega)|^2$ mean the reflection and transmission coefficients, respectively, whereas $\mathcal{A} = (|A_{11}(\omega)|^2 + |A_{12}(\omega)|^2) = (1 - \mathcal{R} - \mathcal{T})$ represents the coefficient of absorption.

The elements of the characteristic transformation and absorption matrices of arbitrary four-port device can be calculated in principle from its Green function. However, in case the device has a layered structure of the certain refractive indices, then these elements might be obtained analytically step by step without explicitly calculating the multislab Green function [6]. The latter method seems to be particular convenient for the Bragg reflectors which have periodic structures.

3. Quantum-state transformation

The operator input-output relations (1) determine the transformation that relates the state of the outgoing field to the state of the incoming field [7]. For the further convenience, we will describe the state transformation in the phase-space representation in terms of characteristic functions. In case the signal field is sent to the first port, the normally-ordered input and output characteristic functions then read as

$$\chi^{\mathcal{N}}_{in1}(\beta) = \mathrm{Tr}\{\hat{\rho}\exp(\beta\hat{a}^{\dagger}_{in1})\exp(-\beta^{*}\hat{a}_{in1})\}, \quad \chi^{\mathcal{N}}_{out1}(\beta) = \mathrm{Tr}\{\hat{\rho}\exp(\beta\hat{a}^{\dagger}_{out1})\exp(-\beta^{*}\hat{a}_{out1})\}, \tag{3}$$

in which $\hat{\rho}$ is the density matrix of the whole system and β is a complex variable designating a point in the phase space. Taking into account the operator relation (1), we arrive at the following form for the output characteristic function

$$\chi^{\mathcal{N}}_{out1}(\beta) = \chi^{\mathcal{N}}_{in1}(T^{*}_{11}\beta)\chi^{\mathcal{N}}_{in2}(T^{*}_{12}\beta)\chi^{\mathcal{N}}_{g+}(A^{*}_{11}\beta)\chi^{\mathcal{N}}_{g-}(A^{*}_{12}\beta) \tag{4}$$

given in terms of normally-ordered characteristic functions of the input radiation fields $\chi^{\mathcal{N}}_{in1,2}$ and of device excitations $\chi^{\mathcal{N}}_{g\pm}$ scaled by the corresponding elements of the characteristic transformation and absorption matrices.

In order to determine the quantum closeness of incident and reflected radiation fields, we use the fidelity which may serve as the measure of the quantum reflection efficiency. If the incident field is in a pure quantum state, in terms of the normally-ordered characteristic functions the fidelity can be represented as [8]

$$\mathcal{F} = \frac{1}{\pi}\int d^2\beta \chi^{\mathcal{N}}_{in1}(-\beta)\chi^{\mathcal{N}}_{out1}(\beta)\exp(-|\beta|^2). \tag{5}$$

4. Temperature effects

To find the influence of the mirror environment on the quantum reflection efficiency, we ought to specify the quantum states of the mirror excitations. As soon as we are interested in the temperature effects, we have to assume that the mirror excitations are in chaotic states at certain temperature T. It means that their normally-ordered characteristic functions are

$$\chi^{\mathcal{N}}_{g_\pm}(\beta) = \exp(-\overline{n}_{th}|\beta|^2), \tag{6}$$

where $\overline{n}_{th} = [\exp(\hbar\omega/k_B T) - 1]^{-1}$ denotes the mean number of the mirror excitations. On purpose to eliminate an active interference of the second port in the reflection process from the first one, we consider the radiation field $\hat{a}_{in2}$ to be in the vacuum state, so that its normally-ordered characteristic function is simply to be

$$\chi^{\mathcal{N}}_{a_{in2}}(\beta) = 1. \tag{7}$$

Then the fidelity (5) takes the form

$$\mathcal{F}_{th} = \frac{1}{\pi}\int d^2\beta \chi^{\mathcal{N}}_{in1}(-\beta)\chi^{\mathcal{N}}_{in1}(\sqrt{\mathcal{R}}\beta)\exp\{-(1+\overline{n}_{th}\mathcal{A})|\beta|^2\} \tag{8}$$

that reveals the temperature dependence only in the case of an absorbing mirror with the certain coefficient of absorption $\mathcal{A}$.

5. Results

As an example, we consider a Bragg mirror consisting of repeated pairs of titanium dioxide and silica. The number of the stacks we set to be $N = 12$. For the qualitative analysis, we neglect the dependence of the refractive indices of temperature and fix them as $n_1 = 2.5 + i3\cdot 10^{-2}$ for TiO_2 and $n_2 = 1.5$ for SiO_2. The widths of the pair slabs L_1 and L_2 in the stack are chosen to fulfill the Bragg condition for the center wavelength of the stop band as $L_1/L_2 = \mathrm{Re}(n_2/n_1) = 1.5/2.5$. The behavior of the coefficients of reflection and absorption for such a mirror as functions of the scaled wave number is given in figure 2. The shape of the curve for the reflection coefficient is seen to demonstrate the presence of a stop band.

We then study two cases of quantum states of the signal field which are exemplary for optical processes, namely, coherent and Fock states.

The choice of the incident field in the coherent state $|\alpha_0\rangle$, which can be realized by laser, sets its normally-ordered characteristic function to be

$$\chi^{\mathcal{N}(coh)}_{a_{in1}}(\beta) = \exp(\beta\alpha_0^* - \beta^*\alpha_0). \tag{9}$$

On substituting the expression (9) into equation (8), we obtain that the fidelity of the coherent field reflection from the mirror at the temperature T is represented in the form

$$\mathcal{F}_{coh} = \frac{1}{1+\overline{n}_{th}\mathcal{A}}\exp\left\{-\frac{(1-\sqrt{\mathcal{R}})^2}{1+\overline{n}_{th}\mathcal{A}}|\alpha_0|^2\right\}. \tag{10}$$

Equation (10) shows that thermal noises decrease the fidelity of the coherent field reflection from an absorbing mirror. Moreover, this decrease becomes more pronounced for the intense laser

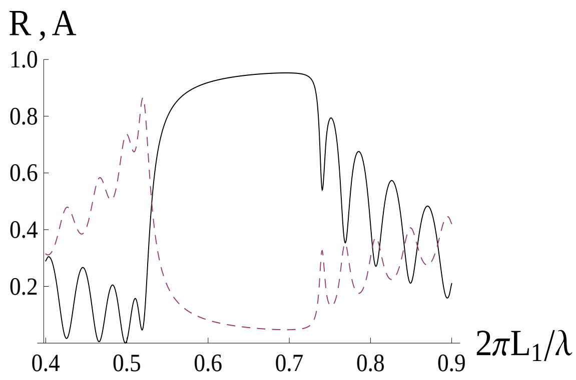


Figure 2. Coefficients of reflection $\mathcal{R}$ (solid) and absorption $\mathcal{A}$ (dashed) as functions of the radiation wave number $2\pi/\lambda$ scaled by the width L_1.

beam, i.e. when $|\alpha_0|^2$ is rather large. It leads to an effective reduction of the mirror stop band. Figure 3 illustrates such a behavior of the fidelity of the coherent field reflection on the model parameters.

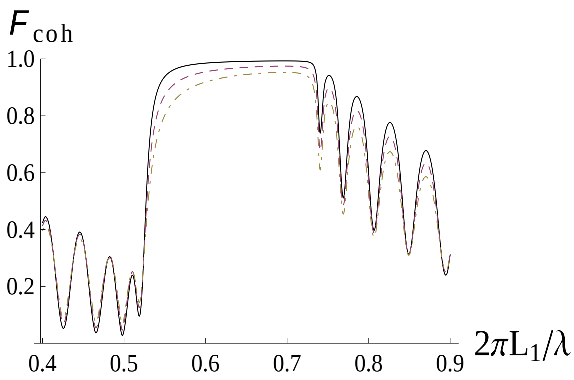
If the incident field is in the n-photon Fock state, its normally-ordered characteristic function takes the form

$$\chi_{a_{in1}}^{\mathcal{N}(Fock)}(\beta) = L_n(|\beta|^2). \tag{11}$$

where L_n is the Laguerre polynomial of the n-th order. Using equation (8), we find that the fidelity of reflection of the n-photon Fock state from the mirror at the temperature T reads as

$$\mathcal{F}_{Fock} = \frac{(\overline{n}_{th}\mathcal{A} - \mathcal{R})^n}{(1 + \overline{n}_{th}\mathcal{A})^{n+1}} P_n\left(1 + \frac{2\mathcal{R}}{(\overline{n}_{th}\mathcal{A} - \mathcal{R})(1 + \overline{n}_{th}\mathcal{A})}\right), \tag{12}$$

where P_n stands for the Legendre polynomial of the n-th order. The fidelity (12) also decreases with an increase of the coefficient of absorption $\mathcal{A}$ and the level of thermal noise $\overline{n}_{th}$, as well as it displays a considerable degradation for large values of a photon number n (see figure 4). It is worth noting that this dependence turns out to be much stronger than that for the coherent state. All this might greatly reduce the quality of the reflected field in the Fock state.



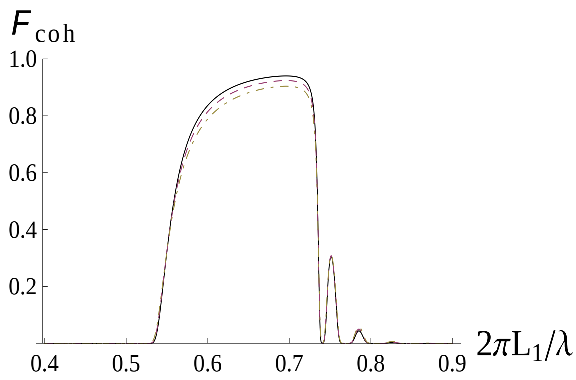


Figure 3. Fidelities of reflection for the coherent states with $\alpha_0 = 2$ (left) and $\alpha_0 = 10$ (right) when the mean number of the mirror excitations $\overline{n}_{th}$ equals 0.1 (solid), 0.5 (dashed), and 1.0 (dot-dashed).

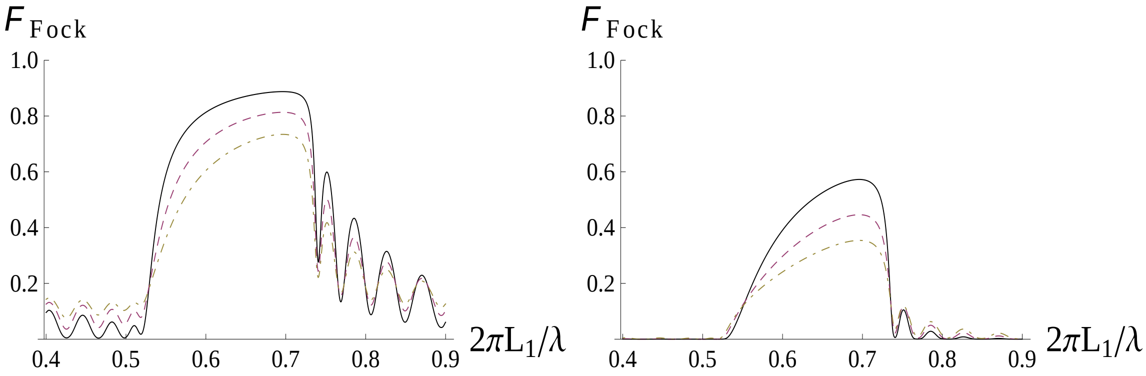


Figure 4. Fidelities of reflection for the Fock states with $n = 2$ (left) and $n = 10$ (right) when the mean number of the mirror excitations $\overline{n}_{th}$ equals 0.1 (solid), 0.5 (dashed), and 1.0 (dot-dashed).

6. Conclusion

In this work, we considered quantum effects of reflection of radiation field states from an absorbing Bragg mirror. Our description of the filed transformation by the Bragg reflector is based on the input-output relations at a four-port device obtained in the Green-function approach for quantization of the electromagnetic field in dispersive and absorbing media. As a measure of reflection, we used the fidelity of incident and reflected states of light. We investigated the dependence of fidelity of reflection with the special emphasis on thermal noises of its environment. Taking as an example coherent and Fock states of the incident light, we found that thermal noises and intensive fields of light might strongly decrease the fidelity of reflection. Moreover, for the highly nonclassical Fock states the decrease of the fidelity is appeared to be very significant. This fact ought to be taken into account when considering optical cavity QED systems.

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