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Some critical remarks on recent results concerning $\mathcal{F}-$contractions in b-metric spaces

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ABSTRACT

This paper aims to correct recent results on a generalized class of $\mathcal{F}-$contractions in the context of $b-$metric spaces. The significant work consists of repairing some novel results involving $\mathcal{F}-$contraction within the structure of b-metric spaces. Our objective is to take advantage of the property $(F1)$ instead of the four properties viz. $(F1)$, $(F2)$, $(F3)$ and $(F4)$ applied in the results of Nazam *et al.* ["Coincidence and common fixed point theorems for four mappings satisfying $(\alpha_s, F)-$contraction", Nonlinear Anal: Model. Control., vol. 23, no. 5, pp. 664–690, 2018]. Our approach of proving the results utilizing only the condition $(F1)$ enriches, improves, and condenses the proofs of a multitude of results in the existing state-of-art.

RESUMEN

Este artículo tiene por objetivo corregir resultados recientes sobre una clase generalizada de $\mathcal{F}-$contracciones en el contexto de $b-$espacios métricos. El trabajo significativo consiste en reparar algunos resultados nuevos que involucran $\mathcal{F}-$contracciones en la estructura de b-espacios métricos. Nuestro objetivo es aprovechar la propiedad $(F1)$ en vez de las cuatro propiedades viz. $(F1)$, $(F2)$, $(F3)$ y $(F4)$ aplicadas en los resultados de Nazam *et al.* ["Coincidence and common fixed point theorems for four mappings satisfying $(\alpha_s, F)-$contraction", Nonlinear Anal: Model. Control, vol. 23, no. 5, pp. 664–690, 2018]. Nuestro enfoque para probar los resultados usando solo la condición $(F1)$ enriquece, mejora y condensa las demostraciones de una multitud de resultados en el estado del arte existente.

Keywords and Phrases: $\mathcal{F}-$contraction, $b-$common fixed point; $b-$metric space; υ_s-complete.

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1 Introduction and preliminaries

Let $\aleph$ be a nonempty set and $\daleth$ be a mapping from $\aleph$ to itself, then the point x from $\aleph$, for which $\daleth x = x$ is called a fixed point of $\daleth$. Note that the fixed point of the map $\daleth$ is also the fixed point of each iteration $\daleth^n$ of the mapping $\daleth$ where n is any natural number. There are examples where the opposite is not true. The existence of a fixed point of a mapping $\daleth : \aleph \to \aleph$ is especially important to examine if the set $\aleph$ is supplied with some kind of distance $\bigwedge : \aleph \times \aleph \to [0, +\infty)$ or by some topology τ. Then, depending on that distance ([28], metrics for example [7]) or topology τ [6, 12, 13], the underlying mapping has one or more fixed points, or does not exist at all. The field that studies fixed points in metric spaces is called metric fixed point theory. If the study of fixed points is performed in topological spaces, then that area is called topological fixed point theory.

If $\daleth$ is a mapping of the metric space $(\aleph, \bigwedge)$ into itself then it is called a contraction if there is a $\lambda \in [0, 1)$ such that for every $x, y \in \aleph$ it holds $\bigwedge(\daleth(x), \daleth(y)) \leq \lambda \cdot \bigwedge(x, y)$. Almost a hundred year ago, S. Banach proved the following significant theorem:

Theorem 1.1 ([5]). *Each contraction $\daleth$ on $(\aleph, \bigwedge)$, a complete metric space, has exactly one fixed point. In addition, for each point $x \in \aleph$, the Picard sequence $\daleth^n x$ converges to that fixed point.*

Numerous mathematicians have attempted to propose the generalizations of Banach's theorem since then. These inferences were in two most important ways: either by changing the axioms of the metric space or by taking another condition instead of the right side in the definition of contraction.

In the first mentioned direction, there were generalized metric spaces, for example, b-metric space, dislocated metric space, rectangular metric space, partial metric space, dislocated b-metric space, and in the second direction, new contractions such as Kanann, Chatterjea, Reich, Hardy-Rogers, Ćirić, Boyd, Wong, etc.

In this paper, we will talk about $\digamma-$contractions in b-metric spaces, combined with various types of admissible mappings. We will first note that all types of admissibility in this paper are introduced in the same way as the corresponding ones introduced in [27]. So, putting in the condition,

$$\upsilon_s(r_1, r_2) \geq s^2 \quad \text{implies} \quad \upsilon_s(\Im(r_1), \Im(r_2)) \geq s^2 \quad \text{for all} \quad r_1, r_2 \in \aleph,$$

of [19, Definition 2] $\frac{1}{s^2} \cdot \upsilon_s = \upsilon$, we get that $\Im : \aleph \times \aleph \to [0, +\infty)$ is υ-admissible introduced in the sense of [27]. In the same way we get that the conditions given in [19, Definitions 2, 3, 4, 5, 6, 7, 8, 9, 10 and 12] can be reduced to the corresponding $\upsilon-$conditions considered in the setting of metric spaces. For further details see [21, 27].

It should be noted that A. I. Bakhtin [4] introduced the idea of b-metric spaces and later considered by S. Czerwik [9]. In fact, the axiom of a triangle in metric spaces is generalized by adding a coefficient $s \geq 1$ on the right-hand side, *i.e.*, $\bigwedge(x,z) \leq s[\bigwedge(x,y) + \bigwedge(y,z)]$ for all $x,y,z \in \aleph$, where $\bigwedge : \aleph \times \aleph \to [0,+\infty)$. Otherwise, there are significant differences between b-metric and ordinary metric. First, it does not have to be a continuous function with two variables such as metric, an open sphere does not have to be an open set. Note that convergence, Cauchyness and continuity of the mapping are defined in the same way as for metric spaces. Also, a convergent sequence can has only one limit value.

Generalizing Banach's principle of contraction [5], D. Wardowski [33] presented the notion of $\digamma-$contraction and manifested a new generalized result as a substitute of Banach's theorem.

Definition 1.2 ([33])**.** *Let $\Gamma : (0,+\infty) \to (-\infty,+\infty)$ be a mapping persuading the assertions described below:*

$(F1)$ *For all $\Upsilon_1, \Upsilon_2 \in (0,+\infty)$ if $\Upsilon_2 > \Upsilon_1$ implies $\Gamma(\Upsilon_2) > \Gamma(\Upsilon_1)$, that is, Γ is strictly increasing function in $(0,+\infty)$;*

$(F2)$ *If $\{\Upsilon_n\}_{n\in\mathbb{N}}$ is a positive sequence of real numbers, then the following is contented:*

$$\lim_{n\to+\infty} \Upsilon_n = 0 \quad \textit{if and only if} \quad \lim_{n\to+\infty} \Gamma(\Upsilon_n) = -\infty;$$

$(F3)$ $\lim_{t\to 0^+} t^{\lambda}\Gamma(t) = 0$, *where* $\lambda \in (0,1)$.

$\mathfrak{F}_\Gamma$ is the set of all functions that satisfy $(F1) - (F3)$.

The following functions $\Gamma_i : (0,+\infty) \to (-\infty,+\infty)$ are in $\mathfrak{F}_\Gamma$: $\Gamma_1(t) = \ln t$; $\Gamma_2(t) = t + \ln t$; $\Gamma_3(t) = -t^{-\frac{1}{2}}$; $\Gamma_4(t) = \ln\left(t+t^2\right)$. For further details on $\mathfrak{F}_\Gamma$ the reader can see [35, 36].

Definition 1.3 ([33])**.** *A mapping $\daleth : \aleph \to \aleph$ is termed as $\digamma-$contraction in the context of metric space $(\aleph, \bigwedge)$ if there exist $\Gamma \in \mathfrak{F}_\Gamma$ and $\tau > 0$ such that for all $\curlywedge, \Upsilon \in \aleph$,*

$$\bigwedge(\daleth(\curlywedge), \daleth(\Upsilon)) > 0 \quad \textit{implies} \quad \tau + \Gamma\left(\bigwedge(\daleth(\curlywedge), \daleth(\Upsilon))\right) \leq \Gamma\left(\bigwedge(\curlywedge, \Upsilon)\right). \tag{1.1}$$

Theorem 1.4 ([33])**.** *If $(\aleph, \bigwedge)$ is a complete metric space and let $\daleth : \aleph \to \aleph$ be an $\digamma-$contraction in the sense of Wardowsski. Then $\daleth$ possesses one and only one fixed point $\curlywedge^* \in X$. On the other hand, the sequence $\{\daleth^n \curlywedge\}_{n\in\mathbb{N}}$ converges to $\curlywedge^*$ for every $\curlywedge \in \aleph$.*

In [8], the authors introduce the following condition,

$(F4)$ If $(\curlywedge_n) \subset (0,+\infty)$ is a sequence such that $\tau + \Gamma(s \cdot \curlywedge_n) \leq \Gamma(\curlywedge_{n-1})$ for every $n \in \mathbb{N}$ and for some $\tau > 0$, then $\tau + \Gamma(s^n \cdot \curlywedge_n) \leq \Gamma\left(s^{n-1} \cdot \curlywedge_{n-1}\right)$, for all $n \in \mathbb{N}$.

$\mathfrak{F}_{\Gamma_s}$ stands for the family of all functions $\Gamma : (0,+\infty) \to (-\infty,+\infty)$ that satisfy $(F1)$, $(F2)$, $(F3)$ and $(F4)$.

Remark 1.5. *It is easy to verify that the condition (F4) implies $b-$Cauchyness of the sequence $\{r_n\}_{n\in\mathbb{N}}$. In other words, this condition is quite strong, but fortunately it can be avoided. We will not use it in our approach. It is therefore superfluous in the whole paper [19].*

The authors in [19] introduce and prove the following:

Let a b-metric space $(\aleph, \bigwedge, s \geq 1)$ be equipped with self-mappings $\hbar, \beth, \Im, \daleth : \aleph \to \aleph$, and υ_s be defined as in [19, Definition 2]. Then they define the next two sets of real numbers:

$$\gamma_{\hbar,\beth,\upsilon_s} = \left\{(\upsilon, \varrho) \in \aleph \times \aleph : \upsilon_s\left(\Im(\upsilon), \daleth(\varrho)\right) \geq s^2 \text{ and } \bigwedge\left(\hbar(\upsilon), \beth(\varrho)\right) > 0\right\} \tag{1.2}$$

and

$$\mathcal{M}_1(\upsilon, \varrho) = \max\left\{\bigwedge\left(\Im(\upsilon), \daleth(\varrho)\right), \bigwedge\left(\hbar(\upsilon), \Im(\upsilon)\right), \bigwedge\left(\beth(\varrho), \daleth(\varrho)\right), \frac{\bigwedge\left(\Im(\upsilon), \beth(\varrho)\right) + \bigwedge\left(\hbar(\upsilon), \daleth(\varrho)\right)}{2s}\right\}. \tag{1.3}$$

For more synthesis on the results based on $F-$contractions, we refer the reader to the informative and notable articles [10, 11, 16, 17, 18, 19, 20, 21, 22, 24, 26, 29, 30, 31, 32, 33, 34].

Theorem 1.6. *Let $\aleph$ be a non-void set and υ_s as described in (1.2). Let the self-maps $\hbar, \beth, \Im, \daleth$ be υ_s-b-continuous on υ_s-complete b–metric space $(\aleph, \bigwedge, s \geq 1)$ such that $\hbar(\aleph) \subseteq \daleth(\aleph), \beth(\aleph) \subseteq \Im(\aleph)$. Assume that for every pair $(r_1, r_2) \in \gamma_{\hbar,\beth,\upsilon_s}$, there exist $\mathit{\Gamma} \in \mathfrak{F}_{\mathit{\Gamma}_s}$ and $\tau > 0$ with*

$$\tau + \mathit{\Gamma}\left(s \cdot \bigwedge\left(\hbar(r_1), \beth(r_2)\right)\right) \leq \mathit{\Gamma}\left(\mathcal{M}_1(r_1, r_2)\right). \tag{1.4}$$

Assume that the pairs $(\hbar, \Im), (\beth, \daleth)$ are υ_s-compatible and the pairs $(\hbar, \beth)$ and $(\beth, \hbar)$ are rectangular partially weakly υ_s-admissible with respect to $\daleth$ and $\Im$ respectively. Then the pairs $(\hbar, \Im), (g, \daleth)$ have the coincidence point (say) υ in $\aleph$. Moreover, if $\upsilon_s(\Im(\upsilon), \daleth(\upsilon)) \geq s^2$, then υ is a common fixed point of $\hbar, \beth, \Im, \daleth$.

To begin, we will utilize the following two findings to show that certain Picard sequences in b-metric spaces $(\aleph, \bigwedge, s \geq 1)$ are b-Cauchy. The proof is an exact replica of the equivalent result in [14] (see also [1]).

Lemma 1.7. *Let $\{r_n\}_{n\in\mathbb{N}}$ be a sequence in b-metric space $(\aleph, \bigwedge, s \geq 1)$ such that*

$$\bigwedge(r_n, r_{n+1}) \leq \lambda \cdot \bigwedge(r_{n-1}, r_n) \tag{1.5}$$

for some $\lambda \in [0, \frac{1}{s})$ and for each $n \in \mathbb{N}$. Then $\{r_n\}_{n\in\mathbb{N}}$ is a b-Cauchy sequence.

Remark 1.8. *It is worth noting that the preceding Lemma holds for each $\lambda \in [0, 1)$ in the context of b-metric spaces. See [15] for additional information.*

Lemma 1.9. *Let $\{r_n\}_{n\in\mathbb{N}}$ be a Picard sequence in b-metric space $(\aleph, \bigwedge, s \geq 1)$ induced by a mapping $\daleth : \aleph \to \aleph$ and let $r_0 \in \aleph$ be an initial point. If $\bigwedge(r_n, r_{n+1}) < \bigwedge(r_{n-1}, r_n)$ for all $n \in \mathbb{N}$ then $r_n \neq r_m$ whenever $n \neq m$.*

In the succeeding analysis, we make use of the following known lemma [3, 20, 23].

Lemma 1.10. *Suppose that $\{r_n\}_{n\in\mathbb{N}}$ belongs to a metric space $(\aleph, \bigwedge)$ and satisfies $\lim\limits_{n\to+\infty} \bigwedge(r_n, r_{n+1}) = 0$ is not a Cauchy sequence. Then, there exists $\varepsilon_1 > 0$ and sequences of positive integers $\{n_q\}$, $\{m_q\}$, $n_q > m_q > q$ such that each of the sequences,*

$$\bigwedge\left(r_{n_q}, r_{m_q}\right), \bigwedge\left(r_{n_q+1}, r_{m_q}\right), \bigwedge\left(r_{n_q}, r_{m_q-1}\right), \bigwedge\left(r_{n_q+1}, r_{m_q-1}\right), \bigwedge\left(r_{n_q+1}, r_{m_q+1}\right),$$

tends to ε_1^+ when $q \to +\infty$.

Remark 1.11. *Based on $\Gamma(a-) \leq \Gamma(a) \leq \Gamma(a+), a \in (0, +\infty)$, we conclude that $\lim\limits_{a\to b^-} \Gamma(a) = \Gamma(b-)$ and $\lim\limits_{a\to b^+} \Gamma(a) = \Gamma(b+)$. For particular details see [2] and [25].*

Likewise, if $\Gamma : (0, +\infty) \to (-\infty, +\infty)$ is a strictly increasing function, then either $\Gamma(0+) = \lim\limits_{a\to 0^+} \Gamma(a) = m$, $m \in \mathbb{R}$ or $\Gamma(0+) = \lim\limits_{a\to 0^+} \Gamma(a) = -\infty$.

Remark 1.12. *Before giving the proof of Theorem 1.6, we note that some parts of the formulations of all theorems and their consequences are incorrect. For example, "for each $(r_1, r_2) \in \gamma_{\hbar,\beth,\upsilon_s}$ there exist $\Gamma \in \mathfrak{F}_{\Gamma_s}$ and $\tau > 0$ such that $\ldots$". It is evident that it should be "there is $\Gamma \in \mathfrak{F}_{\Gamma_s}$ and $\tau > 0$ such that for all $(r_1, r_2) \in \gamma_{\hbar,\beth,\upsilon_s} \ldots$".*

2 Some improved results

To prove Theorem 1.6, the authors in [19] used all the four properties *viz.* $(F1)$, $(F2)$, $(F3)$ and $(F4)$ of the mapping Γ. In sharp contrast to this practice, in present article we prove the Theorem 1.6 by omitting properties $(F2)$, $(F3)$, $(F4)$ and we make use of $(F1)$ only, *i.e.*, we only require the strict growth of the mapping $\Gamma : (0, +\infty) \to (-\infty. + \infty)$. Additionally, we will distinguish two cases: $s > 1$ and $s = 1$.

Proof. First let $s > 1$.

Since $\Gamma : (0, +\infty) \to (-\infty, +\infty)$ is strictly increasing (satisfies $(F1)$) then inequality (1.4) implies

$$\bigwedge(\hbar(r_1), \beth(r_2)) < \frac{1}{s} \cdot \mathcal{M}_1(r_1, r_2), \tag{2.1}$$

where $\mathcal{M}_1(r_1, r_2)$ is as in 1.3 with $\upsilon = r_1, \varrho = r_2$.

Otherwise the contractive condition (2.1) is well known in the setting of b-metric spaces. The sequence $\{j_n\}$ defined in [19] on page 671 is obviously b-Cauchy according to Lemma 1.7 and Remark 1.5.

Now, let $s = 1$ where $\mathit{\Gamma}$ satisfies only $(F1)$ seems more difficult to prove in Theorem 1.6 than the case with $s > 1$. This is because for $s = 1$ we do not have the condition $\bigwedge(j_n, j_{n+1}) \leq \lambda \cdot \bigwedge(j_{n-1}, j_n)$ for the Picard sequence $\{j_n\}_{n\in\mathbb{N}}$ defined by

$$j_{2n+1} = \hbar(r_{2n}) = \daleth(r_{2n+1}) \text{ and } j_{2n+2} = \beth(r_{2n+1}) = \Im(r_{2n+2}), \tag{2.2}$$

for $\lambda \in [0, 1)$, where $(\aleph, \bigwedge)$ is given metric space. However, if $s = 1$ (in this case $\bigwedge = d$ is a metric) we get that 1.4 implies

$$\tau + \mathit{\Gamma}\left(\bigwedge(j_{2n}, j_{2n+1})\right) \leq \mathit{\Gamma}\left(\bigwedge(j_{2n-1}, j_{2n})\right)$$

$$\text{and } \tau + \mathit{\Gamma}\left(\bigwedge(j_{2n-1}, j_{2n})\right) \leq \mathit{\Gamma}\left(\bigwedge(j_{2n-2}, j_{2n-1})\right), \tag{2.3}$$

for all $n \in \mathbb{N}$, hence follows $\bigwedge(j_n, j_{n+1}) < \bigwedge(j_{n-1}, j_n)$ for each $n \in \mathbb{N}$. So there exist a limit $\delta \geq 0$ of the sequence $\{\bigwedge(j_n, j_{n+1})\}_{n\in\mathbb{N}}$.

If we suppose that this limit $\delta > 0$, then according to the property of the strictly increasing function $\mathit{\Gamma}$ (see Remark 1.8), we get $\tau + \mathit{\Gamma}(\delta+) \leq \mathit{\Gamma}(\delta+)$, which is a contradiction since $\delta > 0$.

Assume to the contrary that $\{j_n\}_{n\in\mathbb{N}}$ is not a Cauchy sequence, according to the Lemma 1.10 and inequality (1.4) with $s = 1, \bigwedge = d, r_1 = r_{2n_q}, r_2 = r_{2m_q-1}$, we get

$$\tau + \mathit{\Gamma}\left(\bigwedge\left(j_{2n_q+1}, j_{2m_q}\right)\right) \leq \mathit{\Gamma}\left(\mathcal{M}_1\left(r_{2n_q}, r_{2m_q}\right)\right), \tag{2.4}$$

where

$$\mathcal{M}_1\left(r_{2n_q}, r_{2m_q}\right) = \max_{q\to+\infty}\left\{\bigwedge\left(j_{2n_q}, j_{2m_q-1}\right), \bigwedge\left(j_{2n_q+1}, j_{2n_q}\right), \bigwedge\left(j_{2m_q}, j_{2m_q-1}\right),\right.$$

$$\left.\frac{\bigwedge\left(j_{2n_q}, j_{2m_q}\right) + \bigwedge\left(j_{2n_q+1}, j_{2m_q-1}\right)}{2}\right\} \to \max\left\{\varepsilon_1^+, 0, 0, \frac{\varepsilon_1^+ + \varepsilon_1^+}{2}\right\} = \varepsilon_1^+. \tag{2.5}$$

By taking the limit in (2.4) with $q \to +\infty$, we acquire

$$\tau + \mathit{\Gamma}\left(\varepsilon_1^+ +\right) \leq \mathit{\Gamma}\left(\varepsilon_1^+ +\right), \tag{2.6}$$

which is a contradiction with $\tau > 0$. Hence, the sequence $\{j_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence.

Until the end of the proof of Theorem 1.6 the function $\mathit{\Gamma}$ will no longer be used.

The continuation of the proof for both cases $(s = 1, s > 1)$ is exactly the same as in [19]. Of course, the application of the function Γ on page 674 in [19] as well as the use of its continuity on the same page is superfluous. Moreover, its continuity is not assumed in the formulation of Theorem 1.6 Also, the uniqueness of the common fixed point for the mappings $\hbar, \daleth, \urcorner$, and $\Im$ follows directly from (2.1) in both cases ($s > 1, s = 1$) without any use of the function Γ. □

Remark 2.1. *For both cases $s > 1$ and $s = 1$ there are different proofs, that the defined sequence $\{j_n\}_{n\in\mathbb{N}}$ is Cauchy. In the second case, the property about the left and right limit of the strictly increasing function Γ is used. Further, one known lemma is used if the sequence in the metric space is not a Cauchy but $\bigwedge(j_n, j_{n+1})$ tends to zero as $n \to +\infty$. The authors in [19] gave one proof for both cases, but applied all four properties of the function Γ. Our approach has improved their method and has shown, as in some already published papers, that $(F1)$ is sufficient to prove a fixed point under many contractive conditions. For the case of two mappings in metric spaces, but with all three properties of Γ, the reader can see [34].*

Remark 2.2. *Theorems 2, 3, 4, 5, 6, 7 and 8 from [19] can be corrected in the same way as Theorem 1 from [19], that is, as Theorem 1.6 in this paper. Of course, only property $(F1)$ can be used in their proofs instead of all four properties in [19]. In their proofs, two cases $s > 1$ and $s = 1$ can be also distinguished.*

We now state a simple example that supports our main result.

Example 1. *Let $X = [0, +\infty)$, $d(x, y) = (x - y)^2$, $Tx = kx$, $k \in [0, \frac{1}{2})$, because obviously $s = 2$. Taking further that $\Gamma(r) = \ln r$, $\tau = 1$, we get that the contractive condition $\tau + \Gamma(s \cdot d(Tx, Ty)) \leq \Gamma(d(x, y))$ is fulfilled whenever $d(Tx, Ty) > 0$.*

3 Conclusions

In this article we have showed, in sharp contrast to published articles, that a reduced set of requirements suffices for the proof of fixed point results regarding generalized class of $\digamma-$contractions in b-metric spaces. By using only the property $(F1)$, instead of the four properties $(F1)$, $(F2)$, $(F3)$ and $(F4)$ used in [19], we were able to produce improved and condesed version of the proofs.

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